

NOTATION FROM SECTIONS 1, 3 AND 4

Notation introduced in §1 and §3 for twisted Foulkes modules

---

$\Delta^{(2^m; k)}$	Set of all elements $\delta$ of the form $\{\{i_1, i'_1\}, \dots, \{i_m, i'_m\}, (j_1, \dots, j_k)\}$ where $\{i_1, i'_1, \dots, i_m, i'_m, j_1, \dots, j_k\} = \{1, \dots, 2m + k\}$
$F$	Ground field of odd characteristic $p$
$\Gamma^{(2^m; k)}$	Set of elements of the form $(\mathcal{S}(\delta), \mathcal{T}(\delta))$ , used only in Lemma 3.3
$H^{(2^m)}$	Foulkes module with permutation basis $\Omega^{(2^m)}$
$H^{(2^m; k)}$	Twisted Foulkes module, $H^{(2^m; k)} = (H^{(2^m)} \boxtimes \text{sgn}_{S_k}) \uparrow_{S_{2m} \times S_k}^{S_{2m+k}}$
$\mathcal{I}(\delta)$	Involution $(i_1, i'_1) \cdots (i_m, i'_m) \in S_{2m+k}$ corresponding to $\delta \in \Delta^{(2^m; k)}$
$K^{(2^m; k)}$	Submodule of $F\Delta^{(2^m; k)}$ spanned by $\delta - \text{sgn}(g)\delta g$ for $g \in S_{\mathcal{T}(\delta)}$ and $\delta \in \Delta^{(2^m; k)}$
$\mathcal{S}(\delta)$	$\{\{i_1, i'_1\}, \dots, \{i_m, i'_m\}\}$ if $\delta$ is as above
$\mathcal{T}(\delta)$	$\{j_1, \dots, j_k\}$ if $\delta$ is as above
$\bar{\omega}$	Image of $\omega \in \Omega^{(2^m; k)}$ under $F\Delta^{(2^m; k)} \rightarrow F\Delta^{(2^m; k)}/K^{(2^m; k)}$
$\Omega^{(2^m)}$	Collection of set partitions of $\{1, \dots, 2m\}$ into $m$ sets each of size 2
$\Omega^{(2^m; k)}$	Subset of $\Delta^{(2^m; k)}$ of elements such that $j_1 < j_2 < \dots < j_k$
$z_j$	$p$ -cycle $(p(j-1) + 1, \dots, jp) \in S_{2m+k}$

---

Main notation used in the proof of Theorem 1.2 in §4.

---

$\mathcal{A}$	Basis for $H^{(2^{mp}; k)}(R_r)$ , defined in proof of Lemma 4.2 to be $\{\bar{\omega} : \omega \in \Omega^{(2^m; k)}, \mathcal{I}(\omega) \in \text{Cent}_{S_{2m+k}}(R_r)\}$
$\mathcal{A}_{2t}$	Subset of $\mathcal{A}$ defined immediately before Lemma 4.2
$\mathcal{B}$	$p$ -permutation basis for $M$ , defined in the second step to be $\{s_\omega \bar{\omega} : \omega \in \Omega^{(2^{tp}; (r-2t)p)}, \mathcal{I}(\omega) \in \text{Cent}_{S_{rp}}(C)\}$
$\mathcal{B}(h)$	Subset of the $p$ -permutation basis $\mathcal{B}$ for $M(R_r)$ , defined in Proposition 4.4
$C$	$\langle z_1 \rangle \times \langle z_2 \rangle \times \cdots \times \langle z_r \rangle$
$D_t$	$C \cap N_{S_{rp}}(Q_t)$
$E_t$	$\langle z_1 z_{t+1} \rangle \times \cdots \times \langle z_t z_{2t} \rangle \times \langle z_{2t+1} \rangle \times \cdots \times \langle z_r \rangle$
$\mathcal{I}_{\mathcal{O}}(\omega)$	Involution induced on the blocks $\mathcal{O}_1, \dots, \mathcal{O}_r$ corr. to $\omega \in \Omega^{(2^{tp}; (r-2t)p)}$ such that $\mathcal{I}(\omega) \in C_{S_{rp}}(R_r)$ , defined before Proposition 4.4
$L, L^+$	Factors of $Q_t$ where $L_t \leq S_{\{1, \dots, 2tp\}}$ and $L_t^+ \leq S_{\{2tp+1, \dots, rp\}}$
$M, M_t$	$H^{(2^{tp}; (r-2t)p)}(R_r)$
$\mathcal{O}_j$	$\{p(j-1) + 1, \dots, jp\}$ , orbit of $z_j$
$P$	Fixed Sylow $p$ -subgroup of $S_{rp}$ having $C$ as its base group and $R_r$ in its center and containing $Q_t$ , defined at the start of the second step
$Q_t$	Sylow $p$ -subgroup of $S_2 \wr S_{tp} \times S_{\{2tp+1, \dots, rp\}}$
$R_r$	Cyclic subgroup $\langle z_1 \cdots z_r \rangle$ with support of size $rp$
$T_r$	$\{t \in \mathbf{N}_0 : tp \leq m, 2t \leq r, (r-2t)p \leq k\}$ : $2t$ is the number of orbits of $R_r$ that permute the set part of elements of $\Delta^{(2^m; k)}$

---