INJECTIVE, SURJECTIVE AND BIJECTIVE FUNCTIONS

Let X and Y be sets and let $f : X \to Y$ be a function.

(A) In pairs or threes decide whether each property below is equivalent to f being surjective.

(1) For all $y \in Y$ there exists $x \in X$ such that $f(x) = y$.
(2) For all $y \in Y$ there exists a unique $x \in X$ such that $f(x) = y$.
(3) For all $y \in Y$ the equation $f(x) = y$ has a solution.
(4) For some $y \in Y$ the equation $f(x) = y$ has a solution.
(5) Every output of f has an input.
(6) When f is drawn as a diagram, every dot in Y has an arrow coming into it.
(7) (**Added**) The range of f is equal to the codomain of f.

(B) In pairs or threes decide whether each property below is equivalent to f being injective.

(1) Different inputs to f always give different outputs.
(2) If $x \neq x'$ then $f(x) \neq f(x')$.
(3) For all $y \in Y$, the equation $f(x) = y$ has at most one solution.
(4) For all $x \in X$ there exists $y \in Y$ such that $f(x) = y$.
(5) For all $x \in X$ there exists a unique $y \in Y$ such that $f(x) = y$.
(6) When f is drawn as a diagram, there is at most one arrow leaving each $x \in X$.
(7) When f is drawn as a diagram, no element of Y has two or more arrows coming into it.

(C) Now suppose that $g : \mathbb{R} \to \mathbb{R}$ is a function. Which of the properties below is equivalent to g being (a) injective, (b) surjective? (c) bijective?

(1) The graph of f meets every horizontal line at most once.
(2) The graph of f meets every horizontal line at least once.
(3) The graph of f meets every vertical line.
(4) (**Added**) The graph of f meets every horizontal line exactly once.