(1) Nelson, Edward, *A proof of Liouville's theorem*, Proc. Amer. Math. Soc. **12** (1961) 995.

A PROOF OF LIOUVILLE'S THEOREM

EDWARD NELSON

Consider a bounded harmonic function on Euclidean space. Since it is harmonic, its value at any point is its average over any sphere, and hence over any ball, with the point as center. Given two points, choose two balls with the given points as centers and of equal radius. If the radius is large enough, the two balls will coincide except for an arbitrarily small proportion of their volume. Since the function is bounded, the averages of it over the two balls are arbitrarily close, and so the function assumes the same value at any two points. Thus a bounded harmonic function on Euclidean space is a constant.

PRINCETON UNIVERSITY

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(2) A manufactured extract from the start of an invented project (of implausible scope), painfully written to show as many faults as possible. Please do not blame the author, and do not read if you are of a nervous disposition.

Prime numbers and cryptography

Prime numbers are essential to crytography, Euclid's famous theorem has held generations of mathematicians spellbound in it's inescapable beauty.

Theorem. (Euclid, 400) There are infinitely many prime numbers, where a prime is a number only divisible by itself and 1. (Throughout this project, number means element of $\mathbb{N}$.)

Proof Let $2, 3, 5, .., p$ be the aggregate of primes up to p, and let

$$q = 2.3.5...p + 1$$

Then q is not divisible by any of the numbers 2, 3, 5, ... p. It is therefore either prime, or divisible by a prime between p and q. In either case there is a prime greater than p, which proves the theorem.

(3) The American Mathematical Monthly publishes expository and short research papers of broad mathematical interest. The standard of writing is usually high. We have online access to all issues up to 2006: see `www.jstor.org/journals/00029890.html`. For example, try reading Kac, Mark, *Can one hear the shape of a drum?*, Amer. Math. Monthly **73** (1966) 1–23.

(4) Hamming, Richard, *Error detecting and error correcting codes*, Bell Systems Technical Journal **31** (1950) 147–160. Available online: `www.lee.eng.uerj.br/~gil/redesII/hamming.pdf`.

An extract from the first page is below.

Error Detecting and Error Correcting Codes

By R. W. HAMMING

1. Introduction

THE author was led to the study given in this paper from a consideration of large scale computing machines in which a large number of operations must be performed without a single error in the end result. This problem of "doing things right" on a large scale is not essentially new; in a telephone central office, for example, a very large number of operations are performed while the errors leading to wrong numbers are kept well under control, though they have not been completely eliminated. This has been achieved, in part, through the use of self-checking circuits. The occasional failure that escapes routine checking is still detected by the customer and will, if it persists, result in customer complaint, while if it is transient it will produce only occasional wrong numbers. At the same time the rest of the central office functions satisfactorily. In a digital computer, on the other hand, a single failure usually means the complete failure, in the sense that if it is detected no more computing can be done until the failure is located and corrected, while if it escapes detection then it invalidates all subsequent operations of the machine. Put in other words, in a telephone central office there are a number of parallel paths which are more or less independent of each other; in a digital machine there is usually a single long path which passes through the same piece of equipment many, many times before the answer is obtained.

In a discussion with last year's MSc students, the following strengths and weaknesses were mentioned. As an exercise, read the paper and see if you agree with them: there is a plenty of room for debate, particularly about the perceived weaknesses.

Strengths: well motivated, lots of interesting real-world examples, and made-up small codes. Clear logical structure, that revisits most topics twice: once from an intuitive point of view then from the geometric point of view. Diagrams and tables break up text. Readable by an intelligent non-expert. Ability to relate to reader.

Weaknesses: some proofs could be shortened, typographic error in statement of triangle inequality, long sentences. No abstract or conclusion.