The Liar Game

Mark Wildon

Guessing Games

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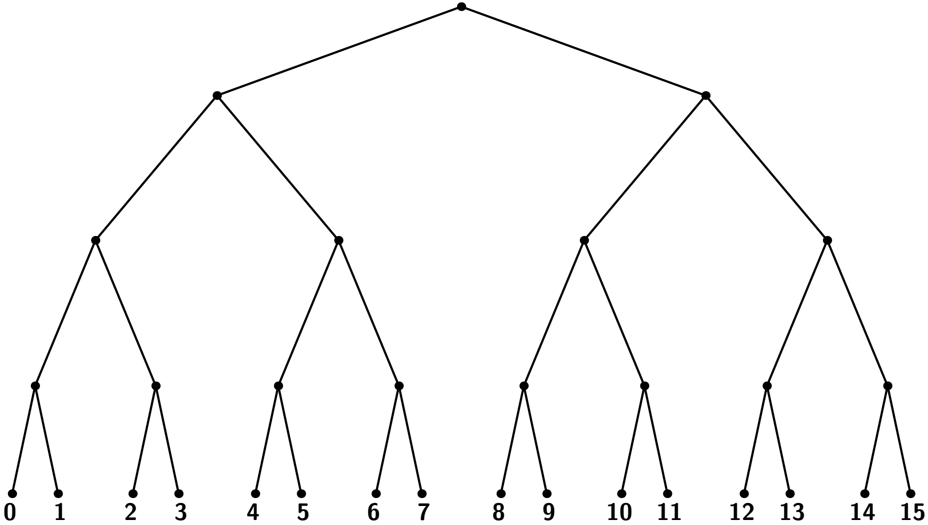

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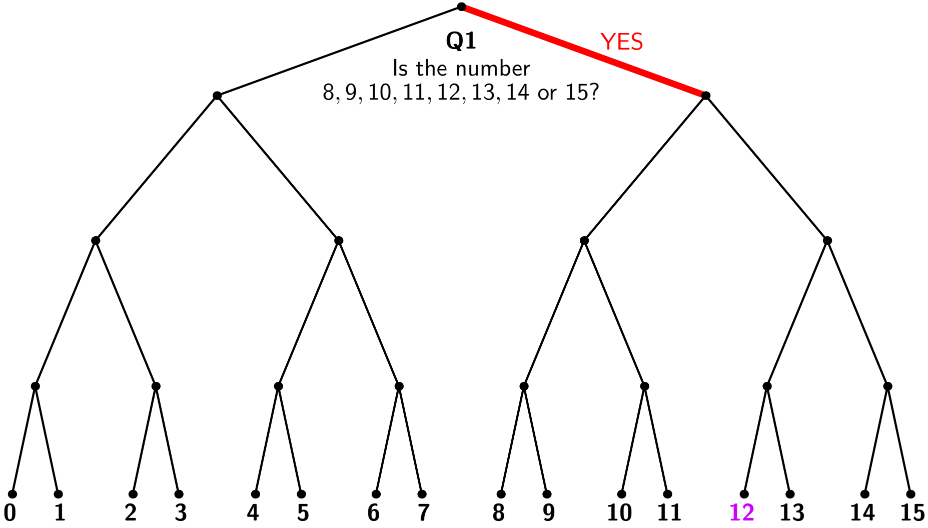

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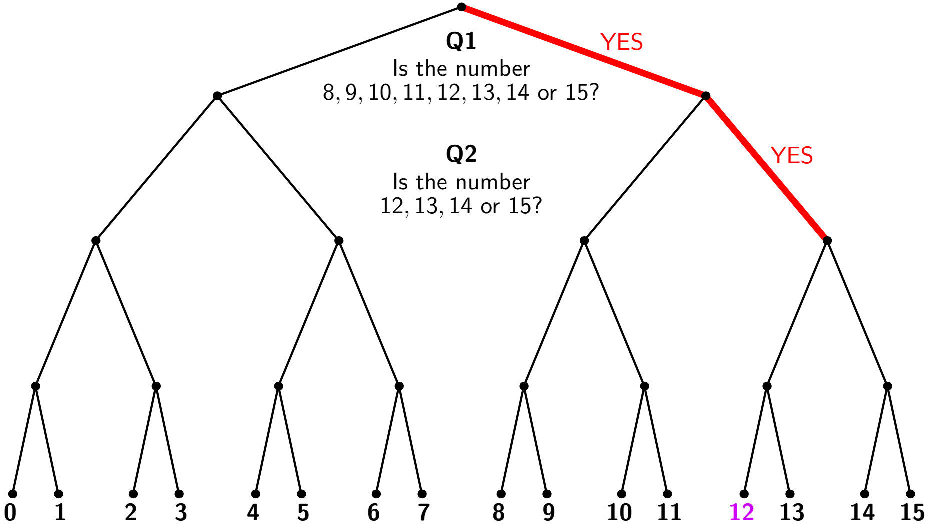

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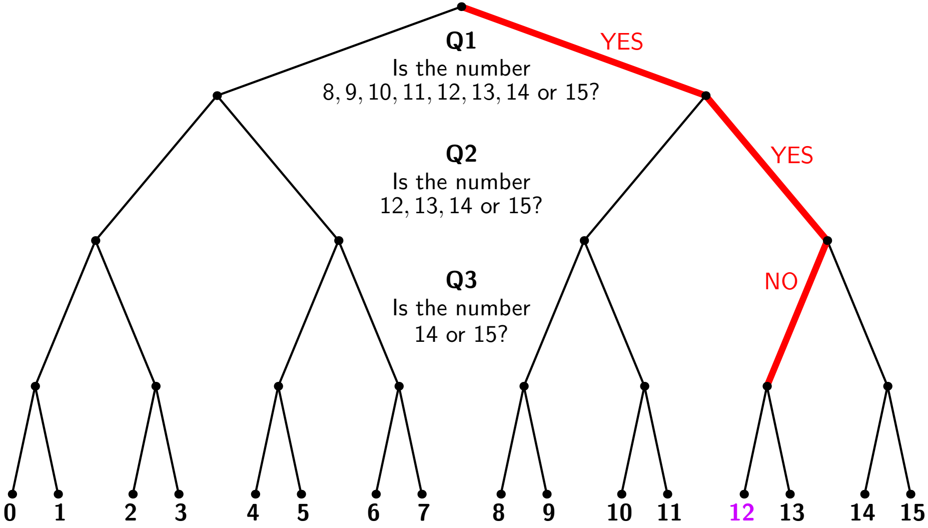

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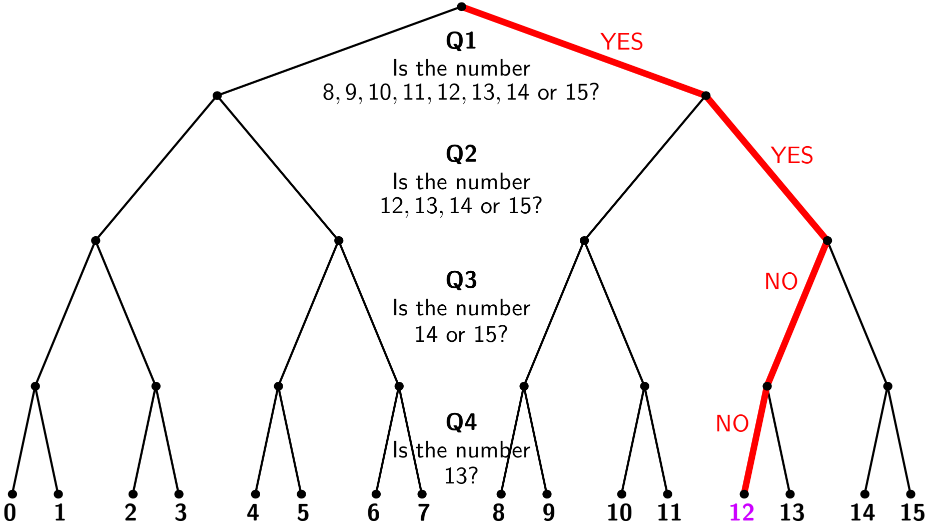

Proofs

Mathematicians like to give rigorous arguments to justify the claims they make.

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- In the worst case, there are at least 4 possible numbers after Question 2.
- In the worst case, there are at least 2 possible numbers after Question 3.

Binary and Computers

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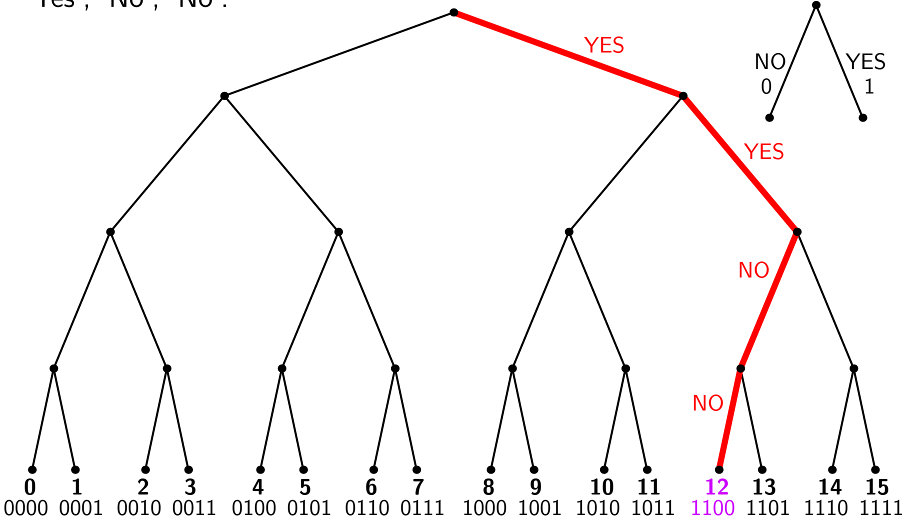

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01110000 11010111 00000100 00010101 11010100 01000110 00000100 11010111
00110101 00000100 01010111 11010111 01110100 00000100 01110100 11010111
00000100 00010101 11010100 00100111 00000100 01110100 00010110 10010100
01110100 00000100 10010110 10110101 00000100 01110100 00010110 11010100
00000100 10110100 11110100 11010100 10110101 01110100 10010110 11010111
01010111 00100111 00000011 11110001 00010110 11010100 01110100 00010110
11010100 00110101 00000100 11000101 01110100 10010110 10110101 00000100
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01010111 01010100 00000100 01110100 11010111 00000100 10110101 11110100
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William Shakespeare (approx 1600)

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10101100 00000000 10101110 00001011 10101100 00101011 01101011 01101001
00001110 00101110 10101100 00101001 00101110 10001101 00100100 00100101
10101100 00101011 01101011 01101001 00001110 00001111 10001000 01001011
01100100 11001010 11001100 11001111 11001111 00001000 00000101 00010100
00001100 00110000 01000000 01011010 00110000 11000010 00110000 00110000
10000000 00011010 00111010 00110000 10000110 10111101 00011010 10101100
00000000 00001011 00101110 10101001 00101011 11101000 10101000 11001011
10001001 10100111 10101001 10101010 11001011 10100101 11001010 01001001
00001110 11001100 11001111 11001111 00001000 00010100 10000001 01011010
00110000 01000101 00010001 01111010 00110000 10100101 01011010 10101100
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Anonymous Microsoft Programmer (2010?)

Part of the machine code for Microsoft Word 2011.

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Why Coding Theory?

A bit gives a single piece of information: ‘NO’ or ‘YES’; ‘on’ or ‘off’; 0 or 1.

- A number between 0 and 15: 4 bits
- A small QR-code:
- Text on this slide
- Full text of *Hamlet*
- Pictures of Royal Holloway
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Errors in reading and writing are inevitable. The 3G specification for mobile phones expects one bit in a thousand to be received wrongly.

Mariner 9

The Mariner 9 probe, launched in 1971, took the first images of Mars. The images were grey-scale, with 64 possible shades of grey for each pixel.

- The pictures were transmitted back to Earth by sending one pixel at a time. Since $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$, each pixel needs 6 bits to send.
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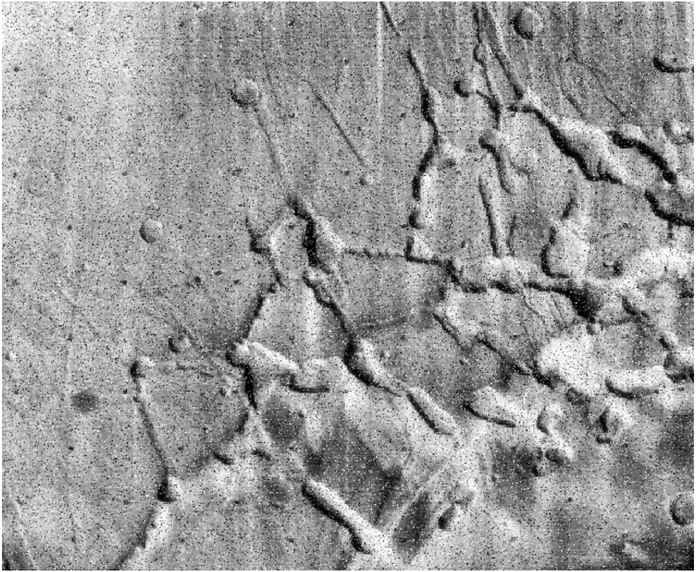
- Encoding each pixel using 6 bits, about 26% of every image would be wrong.
- Instead each pixel was encoded using 32 bits, increasing the length of the transmitted message over five times.

Mariner 9 Image: Improvement Due to Error Correction



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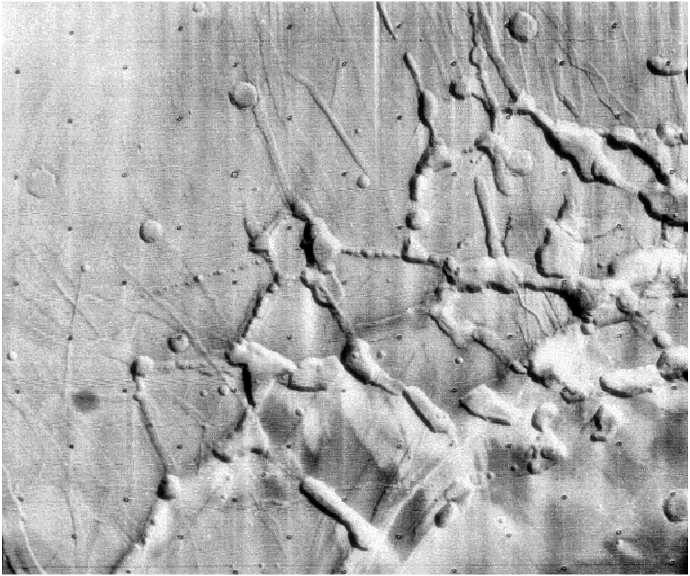

The Mariner 9 Code: 32 of the 64 Mariner 9 codewords

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1\\
0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1\\
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0&0&1&1&0&0&1&1&1&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&0&0&1&1&0&0&1&1\\
0&1&1&0&0&1&1&0&1&0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&0&1&1&0&0&1&1&0\\
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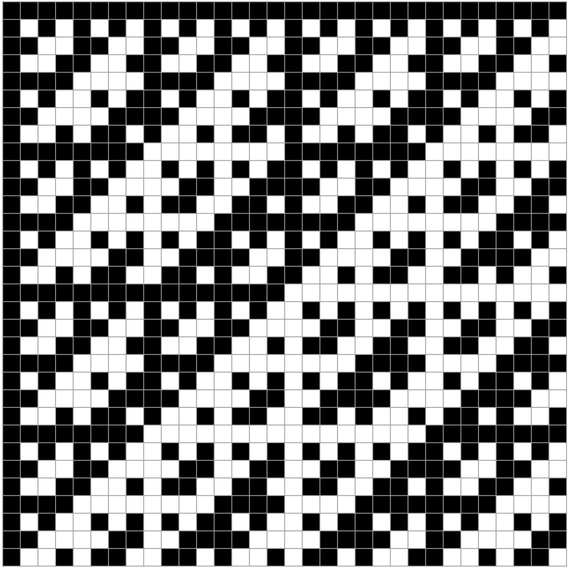
\end{pmatrix}$$

The Mariner 9 Code: 32 of the 64 Mariner 9 codewords



The Liar Game: Dealing with Deliberate Errors

Ask a friend to think of a number between 0 and 15. How many NO/YES questions do you need to ask, if your friend is permitted to lie at most once?

It is not compulsory to lie.

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Coding theory can be used to find a good strategy. Lies correspond to errors in transmission.

The Hamming Code

Richard Hamming discovered a one-error correcting binary code of length 7 with 16 codewords. He invented it because he was fed up with the paper tape reader on his early computer misreading his programs.

It gives an optimal solution to the Liar Game using 7 questions.

Remarkably, it is possible to specify all the questions in advance.

The Hamming Code

Find the binary codeword corresponding to your secret number.

0	0000000	8	1110000
1	1101001	9	0011001
2	0101010	10	1011010
3	1000011	11	0110011
4	1001100	12	0111100
5	0100101	13	1010101
6	1100110	14	0010110
7	0001111	15	1111111

The questions are:

‘Is there a 1 in the first position (far left) of the codeword?’,

‘Is there a 1 in the second position of the codeword?’,

and so on. If there is one lie, then the questioner will write down one wrong bit. But because the Hamming code can correct one error, the questioner can still work out what the number is.

The Square Code

To encode a number between 0 and 15 in the square code

- Write it in binary as $b_1 b_2 b_3 b_4$
- Make a square with these bits.
- Put in four check bits around the edges, computed using modulo 2 arithmetic:

$$0 + 0 = 0, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 1 + 1 = 0.$$

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Imagine you receive this.

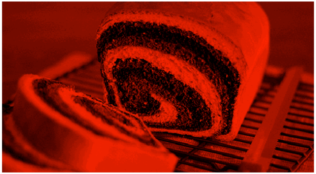
$$\begin{array}{cc|c} 0 & 0 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & \end{array}$$

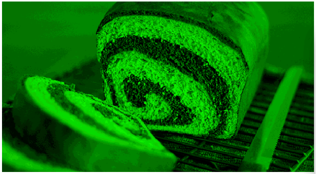
What number do you think was sent?

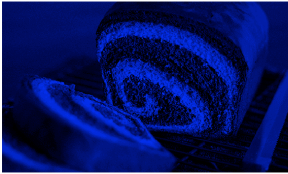
(A) $3 = 0011$ (B) $5 = 0101$ (C) $6 = 0110$ (D) $7 = 0111$

Square Code

Square Code







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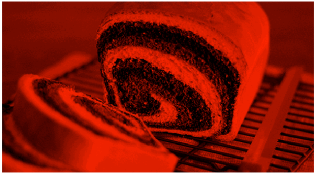
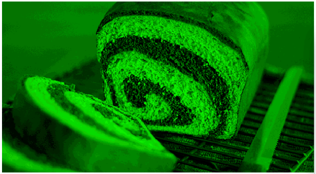
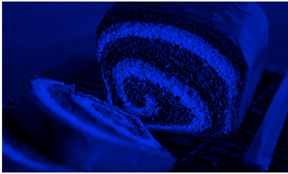

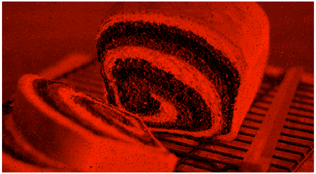
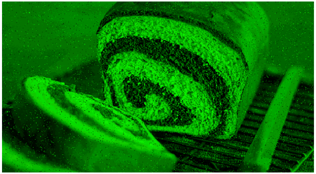
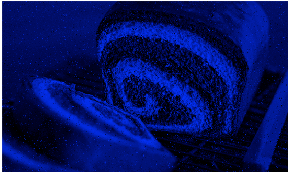

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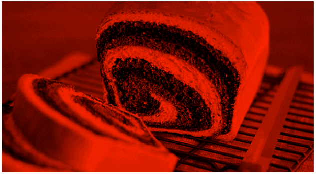

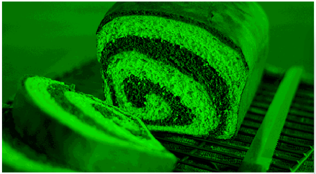

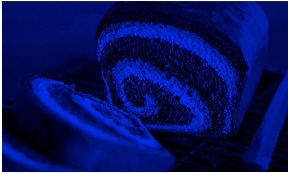

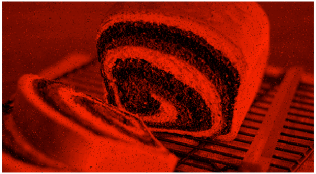

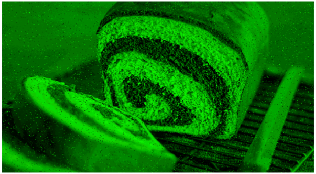

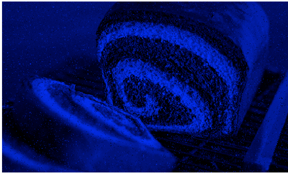

Double Square Code (16 bits) Versus No Coding (4 bits)

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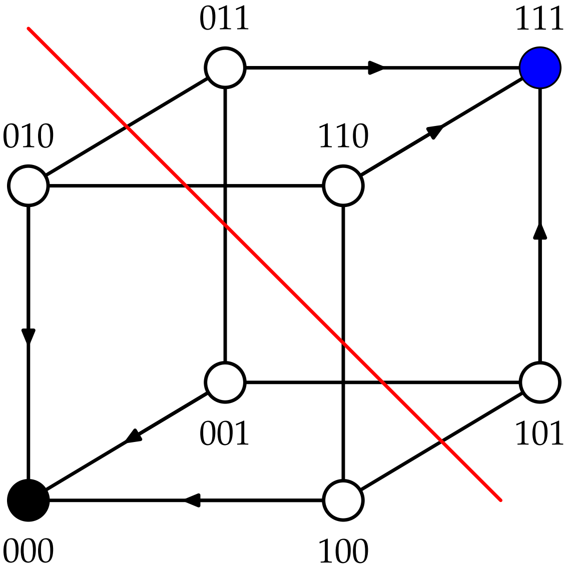
011
111
010
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101
000
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Another Hat Game

You and four friends are lined up. A black or blue hat is put on each person's head. You can see all the hats in front of you, but not your own, or those behind.

So the person at the back of the line can see four hats, the next person can see three, and so on.

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- Why is maths a good subject to study?
- What do maths lecturers do all day?
- How does maths at university differ from A-level maths?
- Are women just as good as men at maths? (**Answer:** Yes!)