# The Liar Game

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## Guessing Games

Ask a friend to thinks of a number between 1 and 15. How many YES/NO questions do you need to ask to find out the secret number?

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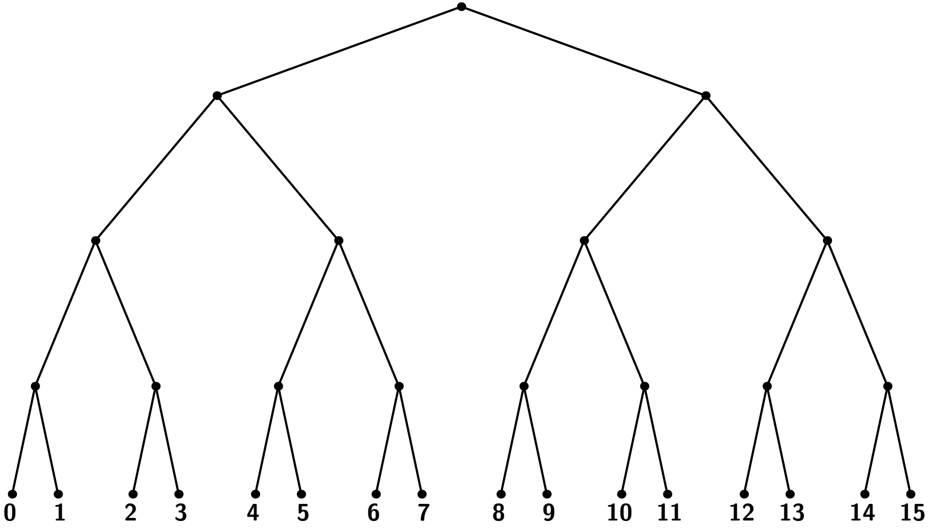

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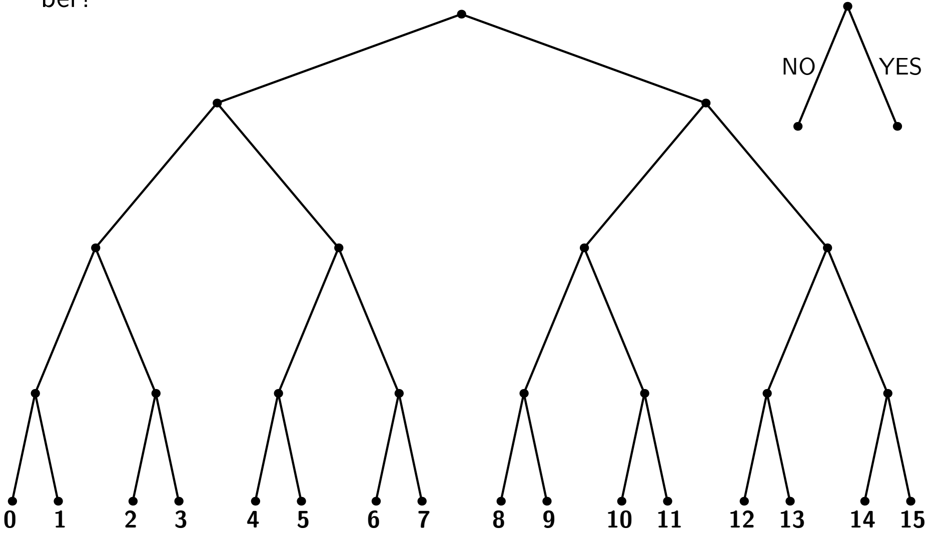

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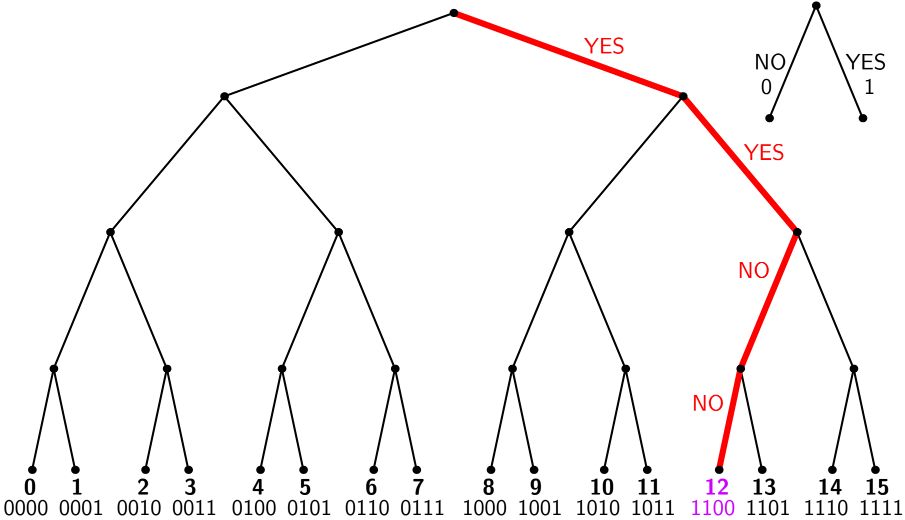

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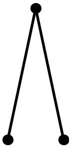

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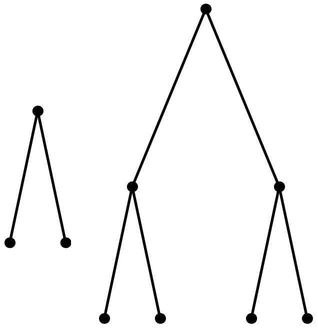

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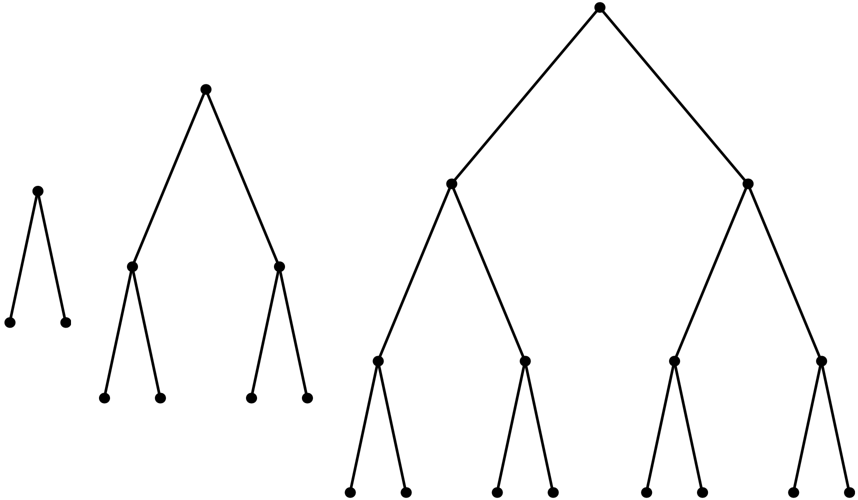

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10101100 00100000 11101010 11101011 00101110 00100000 00101110 11101011
00100000 10101000 00101011 11100100 00100000 00101110 01101000 00101001
00101110 00100000 01101001 10101101 00100000 00101110 01101000 00101011
00100000 00101101 00101111 00101011 10101101 00101110 01101001 11101011
11101010 11100100 11000000 10001111 01101000 00101011 00101110 01101000
00101011 10101100 00100000 10100011 00101110 01101001 10101101 00100000
11101010 11101011 10101000 01101010 00101011 10101100 00100000 01101001
11101010 00100000 00101110 01101000 00101011 00100000 01101011 01101001
11101010 00101010 00100000 00101110 11101011 00100000 10101101 00101111
10101010 10101010 00101011 10101100 11000000 00001110 01101000 00101011
00100000 10101101 01101010 01101001 11101010 10101011 10101101 00100000
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11101010 11100100 11000000 10001111 01101000 00101011 00101110 01101000
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00110000 01110111 01000110 10000000 00011000 00000001 01011101 00011110
10101100 00000000 10101110 00001011 10101100 00101011 01101011 01101001
00001110 00101110 10101100 00101001 00101110 10001101 00100100 00100101
10101100 00101011 01101011 01101001 00001110 00001111 10001000 01001011
01100100 11001010 11001100 11001111 11001111 00001000 00000101 00010100
00001100 00110000 01000000 01011010 00110000 11000010 00110000 00110000
10000000 00011010 00111010 00110000 10000110 10111101 00011010 10101100
00000000 00001011 00101110 10101001 00101011 11101000 10101000 11001011
10001001 10100111 10101001 10101010 11001011 10100101 11001010 01001001
00001110 11001100 11001111 11001111 00001000 00010100 10000001 01011010
00110000 01000101 00010001 01111010 00110000 10100101 01011010 10101100
00000000 00001011 11101010 11101011 01101001 00101110 00101100 00101011
10101001 01101100 00001011 10101111 11101011 01101010 10101010 10101100
00101011 10101110 11001011 10101100 00101011 10101011 00101011 00101110
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Anonymous Microsoft Programmer (2010)

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01100100 11001010 11001100 11001111 11001111 00001000 00000101 00010100
00001100 00110000 01000000 01011010 00110000 11000010 00110000 00110000
10000000 00011010 00111010 00110000 10000110 10111101 00011010 10101100
00000000 00001011 00101110 10101001 00101011 11101000 10101000 11001011
10001001 10100111 10101001 10101010 11001011 10100101 11001010 01001001
00001110 11001100 11001111 11001111 00001000 00010100 10000001 01011010
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00101011 10101110 11001011 10101100 00101011 10101011 00101011 00101110
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*Part of the machine code for Microsoft Word 2011.*

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# Why Coding Theory?

A bit gives a single piece of information: ‘NO’ or ‘YES’; ‘on’ or ‘off’; 0 or 1.

- A number between 0 and 15: 4 bits
- A number between 0 and 1000:
- Full text of *Hamlet*
- Pictures of Royal Holloway (compressed)
- Compact disc of Beethoven 9th
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Errors in reading and writing are inevitable. We can only hope to correct them when they occur.

# A Simple Error Correcting Code

| Number | Encoded as | Number | Encoded as |
|---|---|---|---|
| 0 | 0000 0000 0000 | 8 | 1000 1000 1000 |
| 1 | 0001 0001 0001 | 9 | 1001 1001 1001 |
| 2 | 0010 0010 0010 | 10 | 1010 1010 1010 |
| 3 | 0011 0011 0011 | 11 | 1011 1011 1011 |
| 4 | 0100 0100 0100 | 12 | 1100 1100 1100 |
| 5 | 0101 0101 0101 | 13 | 1101 1101 1101 |
| 6 | 0110 0110 0110 | 14 | 1110 1110 1110 |
| 7 | 0111 0111 0111 | 15 | 1111 1111 1111 |

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**Question.** Suppose you receive 0011 0010 0011. What number was most likely sent?

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| 5 | 0101 0101 0101 | 13 | 1101 1101 1101 |
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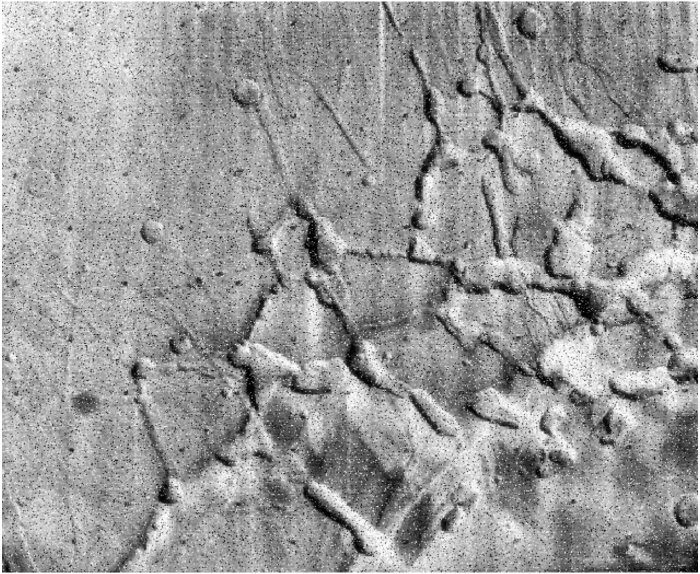
**Question.** Suppose you receive 0011 0010 0011. What number was most likely sent?

**Answer.** Since 0011 0010 0011 differs from 0011 0011 0011 in just once place, it's most likely that the number is 3.

# Mariner 9 Image: Improvement Due to Error Correction



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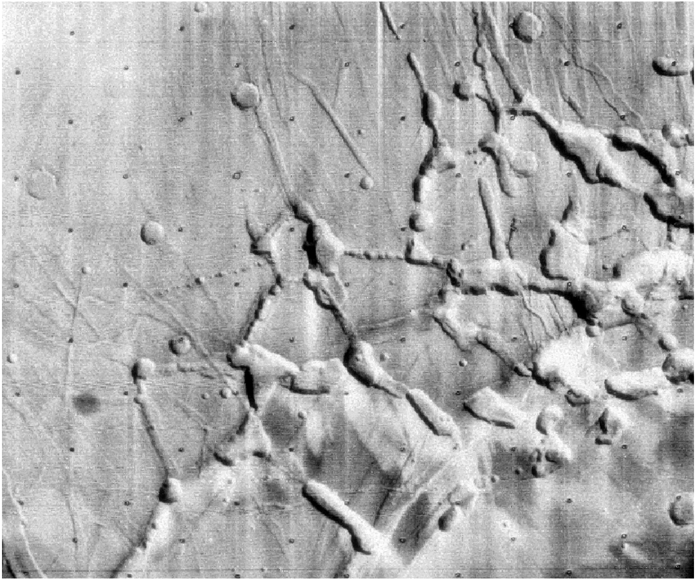
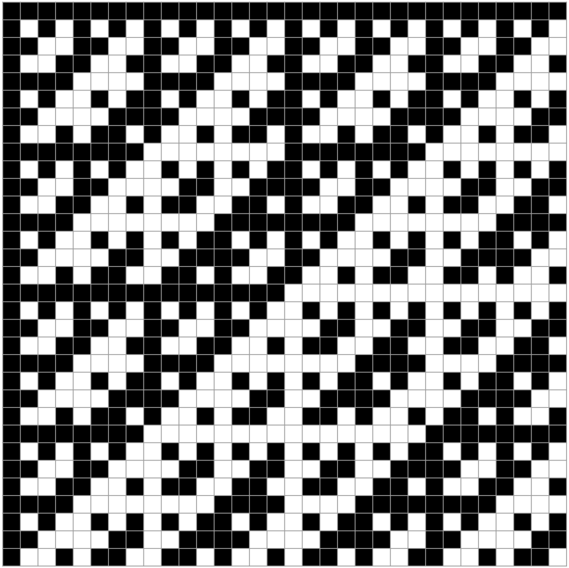


# The Mariner 9 Code: 32 of the 64 Mariner 9 codewords: Black Squares Show 0, White Squares Show 1



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Coding theory gives a seven question strategy. Lies correspond to errors in transmission.

# The Hamming Code

Richard Hamming discovered a one-error correcting binary code of length 7 with 16 codewords. He invented it because he was fed up with the paper tape reader on his early computer misreading his programs.

It gives an optimal solution to the Liar Game using 7 questions.

Remarkably, it is possible to specify all the questions in advance.

# The Hamming Code

Find the binary codeword corresponding to your secret number.

| | | | |
|---|---|---|---|
| 0 | 0000000 | 8 | 1110000 |
| 1 | 1101001 | 9 | 0011001 |
| 2 | 0101010 | 10 | 1011010 |
| 3 | 1000011 | 11 | 0110011 |
| 4 | 1001100 | 12 | 0111100 |
| 5 | 0100101 | 13 | 1010101 |
| 6 | 1100110 | 14 | 0010110 |
| 7 | 0001111 | 15 | 1111111 |

The questions are:

'Is there a 1 in the first position (far left) of the codeword?',

'Is there a 1 in the second position of the codeword?',

and so on. If there is one lie, then the questioner will write down one wrong bit. But because the Hamming code can correct one error, the questioner can still work out what the number is.

Thank you! Any questions?

# A Hat Game Related to Coding Theory

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At the party a black or white hat will be put on each person's head. You can see your friends' hats, but not your own.

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Question: What is a good strategy?

Thank you! Any questions?

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- Why is maths a good subject to study?
- What do maths lecturers do all day?
- How does maths at university differ from A-level maths?
- Are women just as good as men at maths? (**Answer:** Yes!)

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The aim is to find a number between 1 and 15.

- There are **15** possible numbers.

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  - 'Is the number 12? $14\ (\mathrm{NO}) + 1\ (\mathrm{YES}) = 15$
  - 'Is the number prime? $9\ (\mathrm{NO}) + 6\ (\mathrm{YES}) = 15$

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  - 'Is the number prime? $9\ (\mathrm{NO}) + 6\ (\mathrm{YES}) = 15$
- In the worst case there are at least **4** possible numbers after the second question.

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The aim is to find a number between 1 and 15.

- There are **15** possible numbers.
- In the worst case, there are least **8** possible numbers after the first question.
  - 'Is the number 8 or more?' $7\ (\mathrm{NO}) + 8\ (\mathrm{YES}) = 15$
  - 'Is the number even?' $8\ (\mathrm{NO}) + 7\ (\mathrm{YES}) = 15$
  - 'Is the number 12? $14\ (\mathrm{NO}) + 1\ (\mathrm{YES}) = 15$
  - 'Is the number prime? $9\ (\mathrm{NO}) + 6\ (\mathrm{YES}) = 15$
- In the worst case there are at least **4** possible numbers after the second question.
- In the worst case there are at least **2** possible numbers after the third question.

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- In the worst case there are at least **4** possible numbers after the second question.
- In the worst case there are at least **2** possible numbers after the third question.
- So three questions are not enough.