Challenge problems for MT361/MT4561/MT5461
Error Correcting Codes

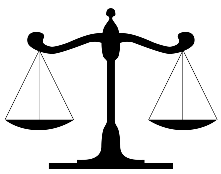
Here are five hopefully interesting problems whose solutions require ideas related to the course.

Problem 1 (Hats I)**.** You and two of your friends are at a party, where a certain game involving hats will be played. The host will place a black or white hat on each person's head. You will be able to see your two friends' hats, but not your own. A few seconds after the hats are in position, the host will shout 'Go!'. Then each of you may either stay silent, or shout 'White' or 'Black'. Everyone who speaks must speak at the same time: you may not wait to hear what your friends say.

- If at least one person speaks, and everyone who speaks shouts the colour of the hat that he or she is wearing, the game ends, and you all get some cake;
- If anyone gets it wrong, or everyone stays silent, the game ends, and there is no cake.

Once the game has started, you may not communicate with your friends in any way. Suggest a good strategy.

Problem 2 (Weighing pennies)**.** You have twelve pennies, one of which *may* be counterfeit, and of a different weight to the other eleven. You also have access to a pair of weighing scales, which can weigh any subset of coins against any other subset. Show how to find the fake penny, if it is present, and identify whether it is heavier or lighter than the other eleven, using just three weighings.



Problem 3 (Eggs)**.** You have been given three Fabergé eggs, identical in every respect, by an owner of a 100 storey building. You are required to find the highest floor from which one of the eggs will survive being dropped. If an egg survives a drop then it is as good as new; if it smashes then it is destroyed for ever. What is the smallest number of drops that are required to be certain of finding the correct answer?

Problem 4 (Hats II)**.** A team of one hundred people are told that in an hour's time they will be taken into a large room, and a hat will be put on each person's head. Each person can see everyone's hat *except* their own. The hats will be of 100 specified colours; it is possible that not every colour will be used.

When a signal is given, every person may say a colour. As usual, everyone speaks simultaneously. If at least one person gets the colour of their hat right, then the team wins. Otherwise the team loses.

Advise the team on a good strategy.

Problem 5 (Candles)**.** At yet another party, the host shows you a cake with eight candles on its top. Above each candle is a robotic arm. When activated, each arm inspects the candle below it, and lights the candle if it is unlit, and snuffs the candle out if it is lit.

You are now locked into a dark cupboard. All you can see are eight switches, which you are told activate the robotic arms. In each turn you may flip any number of the switches. If, after your turn, all the candles are on, you win (and you are allowed to leave the cupboard, and maybe even eat the cake). Otherwise, the host may rotate the cake, changing the correspondence between switches and candles.

Find a strategy that will guarantee that all candles are lit.