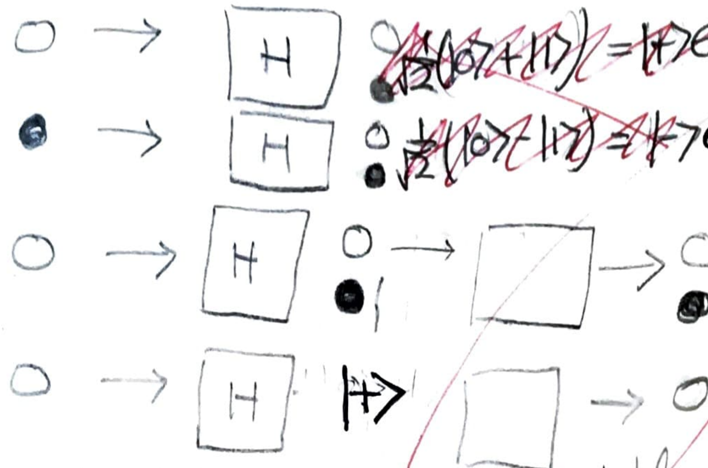
What are quantum states really?

§1 H-box and measurement



~~$\frac{1}{\sqrt2}(|0\rangle+|1\rangle) = |+\rangle \in \mathcal{H}$~~

~~$\frac{1}{\sqrt2}(|0\rangle-|1\rangle) = |-\rangle \in \mathcal{H}$~~

$H|0\rangle = \frac{1}{\sqrt2}(|0\rangle + |1\rangle) = |+\rangle \in \mathcal{H}$

Then Born

$H|1\rangle$

$$H = \frac{1}{\sqrt2}\begin{array}{cc} |0\rangle & |1\rangle \\ \end{array}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = |+\rangle \in \mathcal{H}$$

~~$H|0\rangle = \frac{1}{\sqrt2}(|0\rangle+|1\rangle) \in \mathcal{H}$~~

$$H^2 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

Born rule: $|u\rangle, |v\rangle$ orthonormal basis of $\mathcal{H}$

$\alpha|u\rangle + \beta|v\rangle$ measure in u, v $\begin{cases} |u\rangle & \text{w/prob } |\alpha|^2 \\ |v\rangle & \text{" } |\beta|^2 \end{cases}$

Mathematically very beautiful, but is it how things really are?

§2 Entanglement

$$\mathcal{E} = \frac{1}{\sqrt2}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle)$$

$= \frac{1}{\sqrt2}(|+\rangle\otimes|+\rangle + |-\rangle\otimes|-\rangle)$

$= \frac{1}{2}(|0\rangle\otimes(|+\rangle+|-\rangle) + |1\rangle\otimes(|+\rangle-|-\rangle))$

Alice measures	Alice get (say)	Bob measures	Bob gets
Z basis $|0\rangle, |1\rangle$	$|0\rangle$	Z basis	$|0\rangle$
~~X basis $|+\rangle, |-\rangle$~~	$|+\rangle$	~~X basis~~	$|+\rangle$
Z basis	$|0\rangle$	X basis	$|+\rangle, |-\rangle$ equ prob
Z basis	$|0\rangle$	u, v ($\pi/8$)	$|u\rangle$ with prob $\cos^2\frac{\pi}{8}$; $|v\rangle$ with prob $\sin^2\frac{\pi}{8}$

Difficulty: how can Alice's measurement affect Bob's?

Setup: Alice and Bob are locked in cells 20 light years away.

They share $\begin{cases} \text{an equal (but unknown to them) bit} \in \{0,1\}. \\ \text{an entangled Bell state } \mathcal{E} \end{cases}$

The Gamesmaster tells Alice $b_A \in \{0,1\}$ and Bob $b_B \in \{0,1\}$. Alice and Bob quickly say s_A, s_B. They win iff $s_A + s_B = b_A b_B$.

Thm With optimal strategies

$$\mathbb{P}[\text{Alice, Bob win}] = \begin{cases} 3/4 & \text{classical bit} \\ \cos^2\frac{\pi}{8} \approx 0.854 & \text{qubit} \end{cases}$$

e.g. always say 0

e.g. if Alice hears $b_A = 0$, she says 0; $b_A = 1$, she says 1 & Bob likewise, $\operatorname{corr}(b_A b_B, b_A + b_B) = \frac12$ (prob $3/4$)

Idea: use b_A, b_B to decide which basis to measure in

	things in themselves	observed experience
Plato	ideals	shadows on the cave
Hume	table	sense data
Berkeley	~~X~~	"
Kant	noumena $\boxed{\text{Ding an sich}}$	phenomenon $\boxed{\text{Erscheinung}}$
post Kant	ontology 'existence reality'	epistemology 'states of mind'
	quantum states	epistemic states of subjective belief

Quote overleaf

not just quantum that causes problems: length contracts elec/magnet fields.

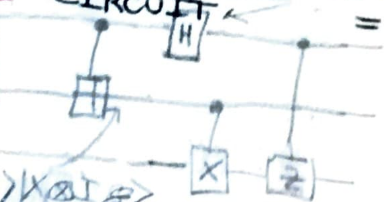
§3 Quantum teleportation $\alpha|+\rangle|\mathcal{E}\rangle + \beta|-\rangle \to |X \otimes I\, \mathcal{E}\rangle$

ZX-CALCULUS
Coecke, ... quantum pictorialism
... 0906.1787

CIRCUIT

$\alpha|0\rangle + \beta|1\rangle = |\psi\rangle$ Alice

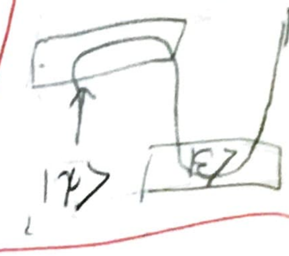
$|\mathcal{E}\rangle$ Bob



$=$ 000 α $|00\rangle|\psi\rangle$
001 β
010 β $+|01\rangle|X\psi\rangle$
011 α
100 α $+|10\rangle|Z\psi\rangle$
101 $-\beta$ $+|11\rangle|XZ\psi\rangle$
110 $-\beta$

$\alpha|0\rangle|\beta\rangle + \beta|1\rangle|X \otimes I\, \mathcal{E}\rangle$

$|X \otimes I\, \mathcal{E}\rangle = |10\rangle + |01\rangle$



HIGH LEVEL

A $|\psi\rangle$
A B $|\mathcal{E}\rangle$ $\longrightarrow$ entangled — A sends 2 bits to B — post A A B $|\psi\rangle$ B

Claim What's really teleported is a state of mind about Bob's qubit, ie teleportation is epistemic not ontic

Dem	Mark	Volunteer	You
Coin1 Heads	2/3	1	1/2
Tails	1/3	0	1/2

flip

measure same/diff on 1, 2.

prepare in front of audience
Alice opp ends blackboard

Coin2 } both Heads 1/2
Coin3 } both Tails 1/2

lie & say same
MUST HAVE STARTED same.

Coin3	same Mark	Volunteer	You
same	$\frac{2}{3}$	1	$\frac{1}{2}$
	$\frac{1}{3}$	0	$\frac{1}{2}$
diff	$\frac{1}{3}$	0	$\frac{1}{2}$
	$\frac{2}{3}$	1	$\frac{1}{2}$

3 coins was H

Analogy: before $\sqrt{\frac{2}{3}}|0\rangle + \sqrt{\frac{1}{3}}|1\rangle$ is Mark's state of knowledge, afterwards it is Bob's

u_θ

$R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

① More simply assuming pure state for new state

$|u_\theta\rangle\langle u_\theta| \otimes I\ (|w\rangle \otimes |w'\rangle) = |u_\theta\rangle\langle u_\theta|w\rangle \otimes |w'\rangle$

$= \langle u_\theta|w\rangle |u_\theta\rangle \otimes |w'\rangle \propto \cancel{|u_\theta\rangle} \otimes |w'\rangle$ w/prob $|\langle u_\theta|w\rangle|^2$

② Apply $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then measure Z

$|w\rangle \otimes |w'\rangle = |{}^x_y\rangle \otimes |w'\rangle \to \left|\begin{matrix} \cos\theta\, x + \sin\theta\, y \\ -\sin\theta\, x + \cos\theta\, y \end{matrix}\right\rangle \otimes |w'\rangle \xrightarrow{|0\rangle} (\cos\theta\, x + \sin\theta\, y)|0\rangle \otimes |w'\rangle$

w/prob $\left|\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \cdot w\right|^2 = |\langle u_\theta|w\rangle|^2$ agrees

Check ① measure in $u_\theta = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ versus ② rotate by $-\theta$ and measure Z: ① $|u_\theta\rangle\langle u_\theta| \otimes I$, $|v_\theta\rangle\langle v_\theta| \otimes I$ as

POVM (chk: $\mathrm{tr}(\rho F_u) = \mathrm{tr}((|w\rangle\langle w| \otimes \rho')F_u) = \mathrm{tr}(|w\rangle\langle u_\theta| \otimes \rho')\langle u_\theta|w\rangle$

$= |\langle u_\theta|w\rangle|^2\, \mathrm{tr}(|u_\theta\rangle\langle u_\theta| \otimes \rho') = |\langle u_\theta|w\rangle|^2\, \mathrm{tr}\,\rho' = |\langle u_\theta|w\rangle|^2$ prove ρ

so by ① $|w\rangle\langle w| \otimes \rho' \to u_\theta \otimes \rho'$ ... $|\langle u_\theta|w\rangle|^2$

$\to |u_\theta\rangle \otimes \rho'$ w/prob $|\langle u_\theta|w\rangle|^2$

Hume: An enquiry concerning human understanding

The table, which we see, seems to diminish, as we remove further from it: But the real table, which exists independently of us, suffers no alteration: It was therefore, nothing but its image, which was present to the mind