Intersective polynomials and Diophantine approximation

It has been known since Vinogradov that for each k, there is an exponent $\theta = \theta(k)$ such that for every positive integer N and real number α, we have $\min_{1 \le n \le N} \|\alpha n^k\| \ll N^{-\theta}$, the bound being uniform in α and N (where $\|\cdot\|$ denotes the distance to the nearest integer). More generally, there is an exponent $\theta = \theta_{k,l}$ such that for any polynomials $f_1, \dots, f_l$ of degree at most k and with zero constant terms, we have $\min_{1 \le n \le N} \max_{1 \le i \le l} \|f_i(n)\| \ll N^{-\theta}$, the bound being uniform in the coefficients of $f_1, \dots, f_l$. Much effort has been put in finding best possible values of θ. In joint work with Craig Spencer, we generalize further the above results, removing the requirement of zero constant terms. It turns out that the only obstructions to the above inequalities are of local nature. We also discuss analogous Diophantine inequalities for polynomials evaluated at primes.