Schauder bases and heights of algebraic numbers

Let $h : \overline{\mathbb{Q}} \rightarrow [0, \infty)$ denote the absolute logarithmic Weil height defined on the field of algebraic numbers. We outline a proof of the following theorem about heights.

Theorem. *Let k be an algebraic number field and α a nonzero algebraic number. Assume that for every $\varepsilon > 0$ there exists a nonzero element β in k and a positive integer m, such that*

$$h\bigl(\alpha^m \beta\bigr) \le \varepsilon m.$$

Then there exists a positive integer n such that α^n belongs to k.

Our proof of this result uses a special Schauder basis in a Banach space determined by the field k. This is joint work with Robert Grizzard.