October 2: **Andrew Treglown** (Birmingham)

**Sum-free sets in the integers**

Abstract: A sum-free set is simply a set of integers which does not contain any solution to $x + y = z$. In this talk we discuss a range of recent developments concerning sum-free sets. We give a sharp count on the number of maximal sum-free subsets in $[n]$, thereby answering a question of Cameron and Erdős. We also consider similar questions in the setting of solution-free sets (that is now we forbid solutions to some other fixed equation). Related complexity results will also be discussed. Underlying all this work is a connection to independent sets in graphs and hypergraphs.

The talk includes joint work with Jozsi Balogh, Hong Liu and Maryam Sharifzadeh; Robert Hancock; Kitty Meeks.

October 9: **Natasha Morrison** (Cambridge)

**The typical structure of sets with small sumset**

Abstract: One of the central objects of interest in additive combinatorics is the sumset $A + B := \{a + b : a \in A, b \in B\}$ of two sets $A, B \subset \mathbb{Z}$. Our main theorem, which improves results of Green and Morris, and of Mazur, implies that the following holds for every fixed $\lambda > 2$ and every $k \geq (\log n)^4$: if $\omega \to \infty$ as $n \to \infty$ (arbitrarily slowly), then almost all sets $A \subset [n]$ with $|A| = k$ and $|A + A| \leq \lambda k$ are contained in an arithmetic progression of length $\lambda k/2 + \omega$. This is joint work with Marcelo Campos, Mauricio Collares, Rob Morris and Victor Souza.

October 16: **Peter Allen** (LSE)

**Packing sparse graphs**
Abstract: Given a family of graphs $G_1, \ldots, G_t$ and a graph $H$, a packing of the $G_i$ into $H$ means a collection of embeddings of the graph $G_i$ into $H$ such that each edge of $H$ is used in at most one embedding. Equivalently, this means a colouring of the edges of $H$ with $t + 1$ colours, where the colour $i$ edges form a graph isomorphic to $G_i$ for each $1 \leq i \leq t$ and the colour $t + 1$ edges are left over.

The question of what families one can pack into a large complete graph, or into a typical large graph, has been quite actively studied recently (in particular motivated by conjectures of Gyarfas and Ringel from the 60s on packing families of trees). We now have quite a good understanding of when one can expect to find an almost-perfect packing (i.e. all but a tiny fraction of edges are used in the packing), and some idea of how to find perfect packings (when all edges are used in the packing). I will outline one approach to these kinds of problem, using a simple probabilistic process.

This is joint work with Julia Boettcher, Dennis Clemens, Jan Hladky, Diana Piguet and Anusch Taraz.

October 23: **Jonathan Chapman** (Manchester)

**Partition regularity and multiplicatively syndetic sets**

Abstract: The study of partition regularity investigates properties of sets which are preserved under finite partitions. For example, if we are given an integer polynomial and a finite colouring of the positive integers, can we always find a monochromatic root? In this talk I will introduce multiplicatively syndetic sets, which are sets of integers with 'bounded multiplicative gaps', and explore their connections with partition regularity. In particular, I will show that a homogeneous integer polynomial has a monochromatic root under any finite colouring if and only if the polynomial has a root over any multiplicatively syndetic set. I will then show how this fact can be used to obtain quantitative bounds for Brauer’s generalisation of van der Waerden’s theorem. I will also mention some open problems concerning minimal multiplicatively syndetic sets.

October 30: **Ehud Meir** (Aberdeen)

**Rings of invariant and the symmetric groups**

Abstract: Let $V$ be a vector space of dimension $n$ over a field $K$ of characteristic zero, and let $T : V \to V$ be a linear endomorphism. By choosing a basis of $V$ we can describe $T$ as an $n \times n$ matrix $(a_{ij})$. While the matrix entries depend on the particular choice of basis, certain polynomials in the variables $a_{ij}$ are invariant to the choice of basis, and provide the most fundamental information about the linear transformation $T$. In fact, we know that $K[a_{ij}]^{GL(V)} = K[c_1, \ldots, c_n]$ where $c_i$ are the coefficients of the characteristic polynomial of $T$. 


In this talk I will describe the ring of invariants \( A := K[\text{End}(V \otimes W)]^{\text{GL}(V) \times \text{GL}(W)}, \)
where \( V \) and \( W \) are two finite dimensional vector spaces. This invariant theory problem arises naturally in the study of finite dimensional Hopf algebras. The representation theory of the symmetric group comes into play here in a fundamental way, and the dimensions of the homogeneous components of \( A \) are given by a formula involving the Kronecker coefficients.

I will then concentrate on the case \( \dim(V) = \dim(W) = 2 \). In this case the Hilbert function of \( A \) was calculated explicitly by Dejan Govc using Mathematica. I will explain these calculations and how they are relate to Zelevinsky’s algebra.

November 6: Kevin Buzzard (Imperial College)

**The future of mathematics?**

Abstract: Over the last few years, something (possibly a mid-life crisis) has made me become concerned about the reliability of modern mathematics, and about how the methods we mathematicians have traditionally used to prove theorems are scaling with the advent of the internet /ArXiv, and pressure on academics to get big results out there. I have started experimenting with a formal computer proof verification system called Lean, integrating it into my undergraduate teaching at Imperial and pushing it to see if it can handle modern mathematical definitions such as perfectoid spaces and the other ideas which got Peter Scholze a Fields Medal in 2018. I personally believe that Lean is part of what will become a paradigm shift in the way humans do mathematics, and that people who do not switch will ultimately be left behind. Am I right? Only time will tell. This talk will be be suitable for a general scientific audience – mathematics undergraduates, computer scientists and philosophers will all find it comprehensible.

November 13: Cong Ling (Imperial College)

**Post-Quantum Cryptography Based on Division Algebras**

Abstract: The Learning with Errors (LWE) problem is the fundamental backbone of modern lattice based cryptography. However, schemes based on LWE are often impractical, so Ring LWE was introduced as a form of structured LWE. Another popular variant, Module LWE, generalizes this exchange by implementing a module structure over a Ring LWE instance. In this work, we introduce a novel variant of LWE over cyclic algebras (CLWE). The proposed construction is both more efficient than Module LWE and conjecturally more secure than Ring LWE, the best of both worlds.

November 20: Kevin Grace (Bristol)

**Templates for Representable Matroids**
Abstract: The matroid structure theory of Geelen, Gerards, and Whittle has led to an announced result that a highly connected member of a minor-closed class of matroids representable over a finite field is a mild modification (known as a perturbation) of a frame matroid, the dual of a frame matroid, or a matroid representable over a proper subfield. They introduced the notion of a template to describe these perturbations in more detail. In this talk, we will define templates and discuss how templates are related to each other. We define a preorder on the set of frame templates over a finite field, and we determine the minimal nontrivial templates with respect to this preorder.

We use templates to obtain results about representability, extremal functions, and excluded minors for various minor-closed classes of matroids, subject to the announced result of Geelen, Gerards, and Whittle. These classes include the class of 1-flowing matroids and three closely related classes of quaternary matroids – the golden-mean matroids, the matroids representable over all fields of size at least 4, and the quaternary matroids representable over fields of all characteristics. This leads to a determination of the extremal functions for these classes, verifying a conjecture of Archer for matroids of sufficiently large rank.

This talk will include a brief introduction to matroid theory and is partially based on joint work with Stefan van Zwam.

November 27: No talk

December 4: No talk

December 11: Joni Teräväinen (Oxford)

Chowla's conjecture at almost all scales

An unsolved conjecture of Chowla states that the Möbius function should not correlate with its own shifts. This can be viewed as a conjecture about the randomness of the Möbius function.

In the last few years, there has been a lot of progress on Chowla's conjecture, which I will survey in the talk. Nearly all of the previously obtained results have concerned correlation sums that are weighted logarithmically, so one wonders whether it is possible to get rid of these weights. We show that one can indeed remove logarithmic weights from previously known results on Chowla's conjecture, provided that one restricts to almost all scales in a suitable sense.

This is joint work with Terry Tao.