Royal Holloway - Mathematics Seminar

Spring term 2020

The Mathematics-seminar takes place on Wednesday at 2pm in McCrea Seminar 2-01 (unless stated otherwise). Here is a campus plan. Tea will be served after the seminar at 3pm next to room 0-08 in the McCrea building. The colloquium talks should be of interest (and accessible) to all members of the Mathematics Department. All are welcome!

January 15: **Ilya Molchanov** (Bern) (Colloquium)

The semigroup of metric measure spaces and its infinitely divisible probability measures

Abstract: A metric measure space is a complete, separable metric space equipped with a probability measure that has full support. Two such spaces are equivalent if they are isometric as metric spaces via an isometry that maps the probability measure on the first space to the probability measure on the second. The resulting set of equivalence classes can be metrized with the Gromov–Prohorov metric. We consider the natural binary operation on this space that takes two metric measure spaces and forms their Cartesian product equipped with the sum of the two metrics and the product of the two probability measures. We show that the metric measure spaces equipped with this operation form a cancellative, commutative, Polish semigroup with a translation invariant metric. There is an explicit family of continuous semicharacters that is extremely useful for establishing that there are no infinitely divisible elements and that each element has a unique factorization into prime elements. We investigate the interaction between the semigroup structure and the natural action of the positive real numbers on this space that arises from scaling the metric. We establish that there is no analogue of the law of large numbers. We characterize the infinitely divisible and stable probability measures and the Lévy processes on this semigroup.

January 22: **Susan Sierra** (Edinburgh)

Enveloping algebras of infinite-dimensional Lie algebras: a survey

Abstract: Given a Lie algebra g, one may form its _universal_enveloping_algebra_, $U(g)$. This is a ring which has the same representation theory as g. Enveloping algebras of finite-dimensional Lie algebras are some of the most well-studied, and thus well-understood, examples in noncommutative ring theory; knowledge of their structure has been important in understanding the representation theory of finite-dimensional Lie algebras and other questions.

In contrast, enveloping algebras of infinite-dimenensional Lie algebras are much more mysterious: for example, it is not known if any such are (left and right) noetherian. We discuss our result (with Walton) that if W is the _Witt_algebra_ of vector fields on the complex affine line, then $U(W)$ is not left or right noetherian: there are one-sided ideals which cannot be finitely generated. We further discuss recent joint work with Petukhov and Iyudu suggesting that all _two-sided_ ideals of $U(W)$ may be finitely generated. We also discuss what is known about enveloping algebras of other infinite-dimensional Lie algebras.

January 29: **Joel Larsson** (Warwick)

Minimum weight spanning structures in randomly weighted graphs

Abstract: Let the complete graph K_n be equipped with i.i.d. edge weights. Given a class of spanning subgraphs, we can ask 'what is the minimum total weight of a subgraph in the class?' This question has received considerable attention for some classes of subgraphs, such as perfect matchings, spanning trees, and Hamilton cycles. In these cases, the minimum weight is known to converge in probability to $\zeta(2), \zeta(3)$ and ≈ 2.04 respectively when the common edge weight distribution is uniform on $[0,1]$. In this talk I will discuss some of the proof ideas, and how to attempt to generalize them to denser spanning subgraphs such as K_t-factors.

February 5: **Radha Kessar** (City Univ. Lond.)

Rationality and finiteness in modular group representation theory

Abstract: Given a ring of coefficients is there a smallest subring over which representation theory is effective? The talk will be an introduction to this question and a survey of recent developments in the context of finite group representations over p-modular coefficient rings.

February 12: **Eamonn O'Brien** (Auckland)

Algorithms for matrix groups

Abstract: How can we compute effectively with a matrix group defined over a finite field? We identify some inherent challenges, and outline a practical model which exploits randomness, geometry and detailed knowledge of the group structure.

February 19: **Ran Levi** (Aberdeen) (Colloquium)

Complexes of Tournaments in Directed Networks

Abstract: Clique graphs whose edges are oriented are referred to in the combinatorics literature as tournaments. We consider a family of semi-simplicial sets, that we refer to as “tournaplexes", whose simplices are tournaments. In particular, given a directed graph G, we associate with it a “flag tournaplex" which is a tournaplex containing the directed flag complex of G, but also the geometric realisation of cliques that are not directed. We define several types of filtration on tournaplexes, and exploting persistent homology, we observe that filtered flag tournaplexes provide finer means of distinguishing graph dynamics than the directed flag complex. We then demonstrate the power of those ideas by applying them to graph data arising from the Blue Brain Project’s digital reconstruction of a rat’s neorcortex. Time permitting we explore connection to another graph invariant.

February 26: **Sasha Kleshchev** (Oregon)

Schur algebras and their representations

Abstract: We will describe Schur algebras as defined and studied first by Schur in his thesis in 1901 and which go back to the 19th century invariant theory. We will discuss some modern developments related to categorification and Broue’s conjecture.

March 4: **Joshua Coyston** (Royal Holloway)

Limit Points of Mahler Measures coming from Digraphs

Abstract: The Mahler measure of a single variable polynomial f can be defined as the geometric mean of |f(z)| on the unit circle. The search to find the smallest Mahler measure, or a complete set of ‘small’ Mahler measures, has been ongoing for almost 100 years. I will introduce a recent concept - the Mahler measure of a directed graph (digraph) - and how some tools from graph theory have helped us find limit points of Mahler measures.

March 11: **Jan-Christoph Schlage-Puchta** (Rostock)

Graph coulourings avoiding rainbow graphs

Abstract: Let $K_n = (G, E)$ be the complete graph with n vertices, $\chi : E \to \{1, \dots, k\}$ an edge colouring of K_n. If G is another finite graph, we say that the colouring χ contains a rainbow-G, if there is an embedding of G into K_n, such that the edges of the image fo G have pairwise different colours. Wagner classified all colourings which do not contain a rainbow-P_6, that is, a path containing 6 vertices. Extending this result to more complex graphs would be extremely tedious. In fact, we showed that for slightly larger graphs G there exist extremely colourful colourings without rainbow-G.

Here we extend this result. We say that $(\alpha, \beta) \in [0,1]^2$ implies a graph G, if every colouring of a sufficiently large K_n, such that there are at least αn vertices which are adjacent to edges of at least βn different colours, contains a rainbow G. We show that the set of pairs implying G is either $[0,1]^2 \setminus \{(0,0)\}$ or a triangle with vertices $(1,0), (1,1), (\beta, 1)$. We classify connected graphs such that $(\beta, 1)$, $0 \le \beta < \frac{1}{2}$ is on the boundary. However, we cannot determine the boundary for the disjoint traingles. In fact, we show that this problem is equivalent to a special case of the Caccetta-Häggkvist conjecture.

March 18: **Eoghan McDowell** (Royal Holloway) !!CANCELLED!!

Counting paths in lattices and a new symmetric polynomial identity

The Lindström–Gessel–Viennot lemma states that the number of tuples of paths in a given lattice is equal to the determinant of a certain matrix. In this talk I will explain the elegant combinatorial argument behind this result, and use it to obtain a new symmetric polynomial identity. This identity generalises both the binomial determinant duality of theoreom of Gessel and Viennot and the symmetric function duality theorem of Aitken.

March 25: **Andrew Thomason** (Cambridge) !!CANCELLED!!