

TITLE: Relative Manin-Mumford for commutative group varieties.

ABSTRACT: The simplest non-trivial case of “Manin-Mumford”, for $\mathbf{C}^* \times \mathbf{C}^*$, shows how to solve a polynomial equation $F(\alpha, \beta) = 0$ in roots of unity α, β ; with various generalizations to torsion points on elliptic curves, abelian varieties, etc. The word “relative” is used for further generalizations referring to families of group varieties such as $E_\lambda \times E_\lambda$ for the Legendre curve defined by $y^2 = x(x-1)(x-\lambda)$. I describe some recent work in this area by Umberto Zannier and myself, and by Harry Schmidt, with proofs based on Bombieri-Pila-Wilkie theory. Together with some properties of additive extensions of elliptic curves, relative Manin-Mumford is one of the main tools allowing progress in Davenport’s Assertion about elementary integration.