

# ENUMERATIVE GALOIS THEORY FOR CUBICS AND QUARTICS

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ABSTRACT: This is joint work with Sam Chow. We consider monic quartic polynomials with integer coefficients and growing box height at most  $H$ . In this setting, we exactly determine the order of magnitude (from above and below) of such polynomials whose Galois group is  $D_4$ . Moreover, we show that  $C_4$  and  $V_4$  polynomials are less frequent than  $D_4$  ones, and that  $D_4, C_4, V_4$  and  $A_4$  polynomials are together less frequent than reducible quartics. Similarly, for integer monic cubic polynomials we show that  $A_3$  cubics are less frequent than reducible cubics. In particular, irreducible non- $S_n$  polynomials are less numerous than reducible ones for  $n = 3$  and  $n = 4$ , for the first time solving two cases (namely degree three and four) of a conjecture by van der Waerden from 1936. A preprint of our work can be found on the ArXiv: <https://arxiv.org/abs/1807.05820>