OPEN PROBLEMS ON ENUMERATING PARTITIONS AND PERMUTATIONS

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Let $p(n)$ denote the number of partitions of n. Erdős and Van Lint have given different elementary arguments showing that $\log p(n) \leq c\sqrt{n}$ where $c = 2\sqrt{\pi^2/6}$. Peter M. Neumann has asked: is there a combinatorial proof, preferably by an explicit bijection, that $\log p(n) = O(\sqrt{n})$? I will outline one approach to this question that uses combinatorial methods arising from the representation theory of the symmetric group. It gives $\log p(n) = O(\sqrt{n^{1+\epsilon}})$ for any $\epsilon > 0$. I will then state some open problems on counting special classes of partitions and permutations. In some the aim is to prove a new result by any method. In others the open question is how sharp a result can be obtained by elementary or entirely combinatorial methods.

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