ALMOST ENGEL COMPACT GROUPS

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We say that a group G is almost Engel if for every $g \in G$ there is a finite set $\mathscr{E}(g)$ such that for every $x \in G$ all sufficiently long commutators $[...[[x, g], g], \dots, g]$ belong to $\mathscr{E}(g)$, that is, for every $x \in G$ there is a positive integer $n(x, g)$ such that $[...[[x, g], g], \dots, g] \in \mathscr{E}(g)$ if g is repeated at least $n(x, g)$ times. (Thus, Engel groups are precisely the almost Engel groups for which we can choose $\mathscr{E}(g) = \{1\}$ for all $g \in G$.)

We prove that if a compact (Hausdorff) group G is almost Engel, then G has a finite normal subgroup N such that G/N is locally nilpotent. If in addition there is a uniform bound $|\mathscr{E}(g)| \leq m$ for the orders of the corresponding sets, then the subgroup N can be chosen of order bounded in terms of m. The proofs use the Wilson–Zelmanov theorem saying that Engel profinite groups are locally nilpotent.

This is joint work with Pavel Shumyatsky.