

**COURSE SPECIFICATION FORM**  
for new course proposals and course amendments

DEPARTMENT OF MATHEMATICS					
<b>Course Code:</b>	MT4860	<b>Course Value:</b>	0.5	<b>Status:</b> (ie:Core, or Optional)	Optional
<b>Course Title:</b>	Permutations and Counting with Groups		<b>Availability:</b> (state which teaching terms)	Term 1	
<b>Prerequisites:</b>	MT2800		<b>Recommended:</b>	MT2830	
<b>Aims:</b>	<p>Recognising the symmetries of mathematical, physical or chemical structures - such as graphs, tilings, the Platonic solids, crystals, molecules or the entire universe - is fundamental to our understanding of these structures. Since symmetries can be described as permutations, we start by studying the basic properties of permutations. By introducing the notion of a group, we are able to capture and investigate algebraically all the symmetries of a given structure. We proceed to develop the basic theory of finite groups, emphasising concrete examples which are often geometrical in nature. By connecting groups back to permutations via group actions, we solve various types of counting problems concerning discrete patterns. Finally, we explore the subgroup structure of finite groups, again based on counting arguments, and we touch upon one of the most striking achievements of 20<sup>th</sup> century mathematics, the classification of finite simple groups.</p>				
<b>Learning Outcomes:</b>	<p>At the end of the course a student should be able to</p> <ul style="list-style-type: none"> <li>• Calculate, use and interpret the cycle structure of permutations;</li> <li>• Understand the concepts of (normal) subgroups and quotient groups;</li> <li>• Apply all the isomorphism theorems;</li> <li>• Count the number of orbits and determine their sizes in specific group actions;</li> <li>• Apply the concept of a group action to count discrete patterns;</li> <li>• Use the Sylow theorems to show that certain finite groups are not simple;</li> <li>• Prove that the alternating groups are in general simple.</li> </ul>				
<b>Course Content:</b>	<p><b>Permutations:</b> sign of a permutation, cycles, transpositions and cycle structure of permutations, symmetric and alternating groups.  <b>Groups:</b> group axioms, subgroups, cosets, Lagrange's theorem, order of a group element, homomorphisms, normal subgroups, quotient groups, isomorphism theorems.  <b>Examples:</b> cyclic groups, permutation groups, e.g. dihedral groups (symmetries of regular polygons) and symmetries of the Platonic solids, matrix groups.  <b>Group actions:</b> definition of a group action, connection with permutations (Cayley's theorem), orbits and stabilizers, the size of an orbit, the number of orbits, application to counting problems concerning discrete patterns (Polya's theorem).  <b>Subgroup structure:</b> conjugacy classes, centralizers, <math>p</math>-groups, Sylow theorems, simplicity of the alternating groups in general, non-simplicity of some groups such as <math>p</math>-groups and <math>pq</math>-groups.</p>				
<b>Teaching &amp; Learning Methods:</b>	<p>33 hours of lectures and examples classes.            117 hours of private study, including work on problem sheets and examination preparation. This may include discussions with the course leader if the student wishes.</p>				
<b>Key Bibliography:</b>	<p>Groups and Symmetry – M A Armstrong (Springer). 512.51 ARM            Groups: a Path to Geometry – R P Burn (Cambridge UP). 512.51 BUR            A First Course in Abstract Algebra with Applications– J.J Rotman (Pearson Prentice Hall). Library Ref.            The Theory of Groups: an Introduction – J.J Rotman (Allyn &amp; Baron). 512.51 ROT</p>				

<b>Formative Assessment &amp; Feedback:</b>	Formative assignments in the form of 8 problem sheets. The students will receive feedback as written comments on their attempts.
<b>Summative Assessment:</b>	<p><b>Exam (%)</b> Four questions out of five in a two-hour paper: 100%</p> <p><b>Coursework (%)</b> None</p> <p><b>Deadlines:</b> n/a</p>

The information contained in this course outline is correct at the time of publication, but may be subject to change as part of the Department's policy of continuous improvement and development. Every effort will be made to notify you of any such changes.