

## *Errata to Introduction to Lie Algebras*

We would like to thank Thorsten Holm for many of the corrections below. The reprinted 1st edition (published June 2007) incorporates all corrections except those marked ( $\dagger$ ).

We are grateful to Michael Andrews for the correction to the proof of Theorem 11.10. We are grateful to Jim Humphreys for pointing out a serious problem with the proof of Theorem 9.16. (See the correction in bold type below.) Fortunately there are no knock-on effects, except to make the intended solution of Exercises 9.14 and 9.15 unviable.

We thank Benjamin Sambale for an extensive list of further important corrections and several suggested improvements, including a simpler solution to Exercise 11.9 and important corrections to Lemma 12.2 and Exercise 19.2.\*

We are grateful to Matthias Franz for pointing out a gap in the proof of Proposition 11.21 and outlining a correction, which we have adopted. We thank David Tiersch for pointing out a mistake in the diagram on page 70, Hassan Azad for a correction to §15.2 and Bonita Graham for a correction to the proof of Lemma 9.8. We are grateful to Andrew Goetz for corrections to mathematical errors in Theorem 9.11, Exercise 11.12 and page 212, corrections to several other minor errors, and for supplying detailed suggestions of a number of other potential improvements to the exposition. We thank Kai Meng Tan for an important correction to §12.1. We thank James Craig for a correction to page 213 in the proof of Weyl's theorem. We thank David Hemmer and Mikko Korhonen for independently pointing out an error in Exercise 7.12(i). We thank Neal Livesay for the correction to the proof of Lemma 10.1(i) and for many corrections to minor errors. We thank Amit Kulshrestha for a correction to the proof of Lemma 13.6. We thank Michael Wemyss for a significant correction to Exercise 2.7. We thank Jun Nakamaru-Pinder for the correction to page 13.

We thank Adelaide Sheard for the correction to the proof of Proposition 13.12.

We thank Joshua Capel for the correction before Theorem 6.3 on page 48.

Please send further corrections or comments to [mark.wildon@rhul.ac.uk](mailto:mark.wildon@rhul.ac.uk).  
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p13, top equation ( $\dagger$ ) There is an error with the primes in the proof that  $[w', z'] + I = [w, z] + I$ . Replace the displayed equation with

$$\begin{aligned}[w', z'] &= [w + (w' - w), z + (z' - z)] \\ &= [w, z] + [w' - w, z] + [w, z' - z] \\ &\quad + [w' - w, z' - z],\end{aligned}$$

p15, Exercise 2.7(i) Replace  $\{(x_2, 0) : x_2 \in L_1\}$  with  $\{(0, x_2) : x_2 \in L_2\}$ .

p15, Exercise 2.7(ii) ( $\dagger$ ) Add the assumption that  $L_1$  and  $L_2$  are non-abelian, and change the conclusion to  $J = 0$ . (Alternatively, interpret ‘proper’ to mean ‘non-zero and proper’: then the conclusion holds as stated, and  $L_1$  and  $L_2$  must be one-dimensional and abelian.)

p15, Exercise 2.7(iii) ( $\dagger$ ) Add the assumption that  $L_1$  and  $L_2$  are non-abelian and remove the assumption that  $L_1$  and  $L_2$  are not isomorphic.

p22, proof of Theorem 3.2 In second display, replace  $[x, w]$  with  $[x, z]$  and  $[y, w]$  with  $[y, z]$ .

p25, line 2 Replace  $\lambda \in L$  with  $\lambda \in \mathbf{C}$ .

p25, Exercise 3.3 ( $\dagger$ ) Add that  $\lambda, \mu, \nu$  are not all zero, to ensure that the Lie subalgebra is 3-dimensional.

p30, Definition 4.6 It should perhaps be emphasised here that we are only considering finite-dimensional Lie algebras.

p32, Section 4.3 ( $\dagger$ ) In ‘as a subalgebra of a Lie algebras of upper triangular matrices.’ replace ‘algebras’ with ‘algebra’.

p33, Exercise 4.2 Replace ‘Show that  $A$  belongs ...’ with ‘Show that  $x$  belongs ...’

p34, Exercise 4.8 ( $\dagger$ ) In (i) replace  $L^3 = 0$  with  $L^2 = 0$  and in (ii) replace  $L^4 = 0$  with  $L^3 = 0$ .

p41, proof of Lemma 5.5 Replace ‘For column  $r$ ’ with ‘For column  $r + 1$ ’. In last line, replace ‘column  $r$ ’ with ‘column  $r + 1$ ’.

p44, Exercise 5.7 (†) In last line, replace  $xy^{m-k}$  with  $y^{m-k}x$ .

p46, Exercise 6.1(ii) Replace  $\bar{x} : V/U \rightarrow V/W$  with  $\bar{x} : V/U \rightarrow V/U$ .

p48, before Theorem 6.3 in ‘for all  $x_0, x_1, \dots, x_m \in V$ ’, replace  $V$  with  $L$ .

p48, proof of Theorem 6.3 Replace  $x \in X$  with  $x \in L$ .

p49, proof of Theorem 6.3 Replace  $\tilde{L}$  with  $\bar{L}$ .

p49, Remark 6.4 Delete the first ‘of’ in the first sentence.

p51, displayed equation Replace  $\lambda(x)$  with  $\lambda(a)$ .

p51, Exercise 6.4 In the last line, replace ‘previous’ with ‘following’.

p54, Example(2) (†) Delete ‘to’ in ‘restriction to of the identity map’.

p57, Example 7.6 Only applies to complex Lie algebras.

p57, after Example 7.6 Replace  $W/V$  with  $V/W$ .

p58, Example 7.8 Replace  $A$  with  $X$  (twice).

p60 The definition of an  $L$ -module homomorphism should read ‘... is a linear map  $\theta : V \rightarrow W$  such that

$$\theta(x \cdot v) = x \cdot \theta(v) \quad \text{for all } v \in V \text{ and } x \in L.$$

p60, second displayed equation Replace ‘ $\theta \circ \varphi_V = \varphi_W \circ \theta$ ’ with ‘ $\theta \circ \varphi_V(x) = \varphi_W(x) \circ \theta$  for all  $x \in L$ ’.

p60, Theorem 7.11 In last line, replace  $V/U$  with  $V/W$ .

p62, Lemma 7.13 (†) Replace  $\theta - \lambda 1_V$  with  $\theta - \lambda 1_S$ ; replace  $S = \ker(\theta - \lambda 1_V)$  with  $S = \ker(\theta - \lambda 1_S)$ ; replace  $\theta = \lambda 1_V$  with  $\theta = \lambda 1_S$ .

p63, Exercise 7.6(iii) Replace ‘every submodule’ with ‘every non-zero submodule’.

p64, Exercise 7.10 (†) In (b), replace  $bv$  with  $v$  since we may assume that  $b = 1$  by (a).

p65, Exercise 7.12(i) (†) The ‘only if’ part of the claimed result is false: a simple counterexample is given by the natural 2-dimensional representation of  $\mathfrak{sl}_2(\mathbf{C})$ . A correct result is as follows: ‘Prove more generally that  $V$  is isomorphic to  $V^*$  if and only if there is a linear isomorphism  $P : V \rightarrow V$  such that

$$P\varphi(x)P^{-1} = -\varphi(x)^{tr}$$

for all  $x \in L$ , where  $\varphi(x) : V \rightarrow V$  is the matrix representing the action of  $x$  on  $V$ .

p65, Exercise 7.12 Part (iii) should be labelled (ii).

p70, Diagram ( $\dagger$ ) Replace  $X^{d-2}Y$  with  $X^{d-2}Y^2$ .

p73, after Corollary 8.6 ( $\dagger$ ) Replace ‘a vector  $v$  of the type considered …’ with ‘a vector  $w$  of the type considered …’.

p74, after Theorem 8.7 Replace ‘Appendix C’ with ‘Appendix B’.

p75, Exercise 8.4 Replace all appearances of  $W$  with  $V$ .

p75, Exercise 8.6 In first line, replace  $L$  with  $\mathfrak{sl}(2, \mathbf{C})$ . Displayed equation should end ‘for  $v \in M$ ’, not ‘for  $v \in V$ ’.

p81, proof of Lemma 9.8 ( $\dagger$ ) Replace  $A_x \circ A_y$  with  $A_x A_y$  and  $\text{tr}(A_x B_x)$  with  $\text{tr}(A_x A_y)$  in final displayed equation.

p82, line 10 Replace  $\dim W \cap \dim W^\perp \neq 0$  with  $W \cap W^\perp \neq 0$ .

p83, proof of Lemma 9.10 ( $\dagger$ ) Change ‘so that there exists  $a \in I'$  to ‘so that there exists a non-zero  $a \in I'$ .

p83, proof of Theorem 9.11 ( $\dagger$ ) It should be justified that  $I$  is simple. If  $J$  is an ideal of  $I$  then  $[I^\perp, J] \subseteq I^\perp \cap I = 0$ , so  $[L, J] = [I, J] = J$ . Hence  $J$  is an ideal of  $L$ , so since  $I$  has minimum non-zero dimension,  $J = I$ .

p84, first displayed equation Replace  $\dots, L_r$  with  $\dots \oplus L_r$ .

p84, line 3 The argument becomes clearer if one replaces  $[I, L_i] \subseteq I \cap L_i = 0$  with  $[I, L_i] \subseteq I \cap I^\perp = 0$ .

p84, Lemma 9.12 ( $\dagger$ ) Change ‘ $I$  is an ideal of  $L$ ’ to ‘ $I$  is a proper ideal of  $L$ ’.

p85 , Prop. 9.13 ] Delete the assumption that  $L$  is finite-dimensional, as this is always in force.

p85, proof of Prop. 9.13 In 3rd paragraph, delete the sentence starting ‘We have ...’ and replace with ‘If  $M$  is properly contained in  $\text{Der } L$  then  $M^\perp \neq 0$ , so it is sufficient to prove that  $M^\perp = 0$ .’

p85, before §9.6 Replace ‘is a direct sum of semisimple Lie algebras’ with ‘is a direct sum of simple Lie algebras’.

**p87, Theorem 9.16(∗)** The argument given before this theorem that the abstract Jordan decomposition of an element  $x$  of a semisimple Lie subalgebra  $L \subseteq \mathfrak{gl}(V)$  agrees with its usual Jordan decomposition as an element of  $\mathfrak{gl}(V)$  is flawed. If the usual Jordan decomposition is  $x = d + n$ , then our argument shows that  $\text{ad } d : \mathfrak{gl}(V) \rightarrow \mathfrak{gl}(V)$  and  $\text{ad } n : \mathfrak{gl}(V) \rightarrow \mathfrak{gl}(V)$  restrict to endomorphisms of  $L$ , and  $\text{ad } d = \text{ad } d'$ ,  $\text{ad } n = \text{ad } n'$ , where  $x = d' + n'$  is the abstract Jordan decomposition of  $x$ . However, we do not know that  $d \in L$ , so we cannot use the uniqueness of the abstract Jordan decomposition to deduce that  $d = d'$ .

The proof of Theorem 9.16 is therefore invalid. The only use made of this theorem later in the book is in the solutions to Exercise 9.14 and Exercise 9.15.

For a correct proof of Theorem 9.16 see J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, volume 9 of Graduate Texts in Mathematics, Springer, New York, 1978, Theorem 6.4. This is [14] in the bibliography.

**p88, after proof (†)** It is incorrect that the result with the incorrect proof is ‘applied several times in the next chapter’. See bold comment above.

**p91, line 3 (†)** Replace ‘Exercise 9.7’ with ‘Exercise 4.7 or 9.7’.

**p93, line 3** Replace  $\alpha : H \rightarrow L$  with  $\alpha : H \rightarrow \mathbf{C}$ .

**p93, line 12** Replace  $\alpha \in L^*$  with  $\alpha \in H^*$ .

**p93, proof of Lemma 10.1(i) (†)** Replace ‘is an eigenvector for each  $\text{ad } h \in H$ ’ with ‘is an eigenvector of  $\text{ad } h$  for each  $h \in H$ ’.

**p94, penultimate paragraph** Replace ‘thus  $H$  is not contained in any larger abelian subalgebra of  $H$ ’ with ‘thus  $H$  is not contained in any larger abelian subalgebra of  $\mathfrak{sl}(3, \mathbf{C})$ ’.

**p97, before §10.4 (†)** Insert ‘the’ before ‘Cartan subalgebra’.

**p98, line 3 (†)** Replace  $\kappa(x, y)$  with  $\kappa(x, w)$ .

- p98, 3rd paragraph Replace  $[h, y] = -\alpha(h)x = 0$  with  $[h, y] = -\alpha(h)y = 0$ .
- p98, Exercise 10.3(ii) Replace  $S_\alpha$  with  $\mathfrak{sl}(\alpha)$ .
- p101, proof of Prop. 10.9, third paragraph Replace the sentence starting ‘We have  $\alpha(v) = 0 \dots$ ’ with ‘The zero-eigenspace of  $h_\alpha$  on  $M$  is  $H$ , which is contained in  $K \oplus \mathfrak{sl}(\alpha)$ . Hence  $v \in (K \oplus \mathfrak{sl}(\alpha)) \cap V = 0$ , which is a contradiction.’
- p101, Prop. 10.10 Replace (ii) with ‘There are integers  $r, q \geq 0$  such that if  $k \in \mathbf{Z}$  then  $\beta + k\alpha \in \Phi$  if and only if  $-r \leq k \leq q$ . Moreover  $r - q = \beta(h_\alpha)$ .’ (The original statement is true, but only the weaker version given above is proved.)
- p102, Proof of Prop. 10.10 ( $\dagger$ ) Related to previous:  $M$  should not be called the root string through  $\beta$  because we are only allowing  $k \in \mathbf{Z}$ .
- p102, after proof ( $\dagger$ ) Replace ‘structure constant’ with ‘structure constants’.
- p104, proof of Corollary 10.13 ( $\dagger$ ) In penultimate line replace  $H \equiv H^*$  with  $H \cong H^*$ .
- p105, proof of Lemma 10.14 The matrix should be transposed; i.e. replace  $(\alpha_1, \alpha_\ell)$  with  $(\alpha_\ell, \alpha_1)$  and  $(\alpha_\ell, \alpha_1)$  with  $(\alpha_1, \alpha_\ell)$ .
- p106, Exercise 10.5 The formula should read  $\dim L = \dim H + |\Phi|$ , not  $\dim L = \dim H + 2|\Phi|$ .
- p106, Exercise 10.6 ( $\dagger$ ) Comma missing in displayed equation.
- p106, Exercise 10.7(i) Replace ‘span  $H$ ’ with ‘span  $H^*$ ’.
- p110, Definition 11.1 Replace ‘A subset  $R$  of a real vector space  $E \dots$ ’ with ‘A subset  $R$  of a real inner-product space  $E \dots$ ’.
- page 110, second displayed equation ( $\dagger$ )  $(t_\alpha, t_\alpha)$  should be  $\kappa(t_\alpha, t_\alpha)$ .
- p111, Exercise 11.1 Change  $E$  to  $\mathbf{R}^{\ell+1}$ .
- p112, Prop. 11.5(b) ( $\dagger$ ) The hypothesis ‘and  $(\beta, \beta) > (\alpha, \alpha)$ ’ is unnecessary. (To prove the more general version, note that if  $\beta$  is shorter than  $\alpha$  then, swapping  $\alpha$  and  $\beta$ , we get that  $\alpha - \beta \in R$ . Then  $\beta - \alpha \in R$  by (R2).)
- p112, Example 11.6(b) ( $\dagger$ ) In penultimate line on this page, replace ‘obtuse angle’ with ‘obtuse or right angle’.
- p113, first diagram The lower root labelled  $\beta$  should be labelled  $-\beta$ .

- p113, Example 11.6(b) After the diagram, replace ‘root space’ with ‘root system’.
- p113, Exampe 11.6(c) ( $\dagger$ ) The  $y$ -coordinates of the four roots  $\beta, \beta + \alpha, \beta + 2\alpha, \beta + 3\alpha$  should be the same.
- p115, §11.3 ( $\dagger$ ) In second line, replace ‘vector space basis for  $R$ ’ with ‘vector space basis for  $E$ ’.
- p117, top line Replace ‘elements of  $\alpha$ ’ with ‘elements of  $B$ ’.
- p117, proof of Thoerem 11.10 ( $\dagger$ ) In line 3, as we do not yet know that  $B$  is a base, it is circular to use Exercise to show that the angle between  $\alpha$  and  $\beta$  is obtuse. Instead note that if the angle is acute, then by Proposition 11.5,  $\alpha - \beta$  is a root, and so either  $\alpha - \beta$ , or  $\beta - \alpha$  lies in  $R^+$ . In the first case  $\alpha = (\alpha - \beta) + \beta$  is the sum of two elements of  $R^+$ , and similarly in the second case for  $\beta$ . This contradicts the definition of  $B$ .
- p122, proof of Proposition 11.21 ( $\dagger$ ) It is not clear that  $\varphi$  satisfies Condition 11.19(b), and in fact it takes some work to show this.

**Step 1:** If  $\alpha \in R$  then we can write  $\alpha = \sum_i k_i \alpha_i$  for some  $k_i \in \mathbf{Z}$ . Then since  $\langle \cdot, \cdot \rangle$  is linear in its first component we have  $\langle \varphi(\alpha), \varphi(\alpha_j) \rangle = \langle \alpha, \alpha_j \rangle$ . This shows that Condition 11.19(b) holds when  $\beta \in B$ .

**Step 2:** Now observe that for any  $\alpha_i, \alpha_j \in B$  we have

$$\begin{aligned} \varphi(s_{\alpha_i}(\alpha_j)) &= \varphi(\alpha_j - \langle \alpha_j, \alpha_i \rangle \alpha_i) \\ &= \varphi(\alpha_j) - \langle \alpha_j, \alpha_i \rangle \varphi(\alpha_i) \\ &= \alpha'_j - \langle \alpha'_j, \alpha'_i \rangle \alpha'_i \\ &= s_{\alpha'_i}(\alpha'_j). \end{aligned}$$

By linearity of  $\varphi$  it follows that  $\varphi(s_{\alpha_i}(v)) = s_{\alpha'_i} \varphi(v)$  for all  $v \in E$ .

**Step 3:** To deal with a general  $\beta \in R$  we use Step 2 and the Weyl group to get around the fact that  $\langle \cdot, \cdot \rangle$  is not linear in its second component. By Proposition 11.14 there exist  $\alpha_{i_1}, \dots, \alpha_{i_r} \in B$  and  $\alpha_j \in B$  such that  $s_{\alpha_{i_1}} \dots s_{\alpha_{i_r}} \alpha_j = \beta$ . By induction on  $r$  it follows from Step 2 that

$$\varphi(\beta) = \varphi(s_{\alpha_{i_1}} \dots s_{\alpha_{i_r}}(\alpha_j)) = s_{\alpha'_{i_1}} \dots s_{\alpha'_{i_r}}(\alpha'_j).$$

Hence for any  $\alpha \in R$  we have

$$\langle \varphi(\alpha), \varphi(\beta) \rangle = \langle \varphi(\alpha), s_{\alpha'_{i_1}} \dots s_{\alpha'_{i_r}} (\alpha'_j) \rangle.$$

For any  $u, v \in E$  and  $\alpha_j \in E$  we have

$$\begin{aligned} \langle u, s_{\alpha_j}(v) \rangle &= \frac{2(u, s_{\alpha_j}(v))}{(s_{\alpha_j}(v), s_{\alpha_j}(v))} \\ &= \frac{2(s_{\alpha_j}(u), v)}{(v, v)} \\ &= \langle s_{\alpha_j}(u), v \rangle. \end{aligned}$$

Hence  $\langle \varphi(\alpha), \varphi(\beta) \rangle = \langle s_{\alpha'_{i_r}} \dots s_{\alpha'_{i_1}} \varphi(\alpha), \alpha'_j \rangle$ . It now follows from Step 2 that

$$\langle \varphi(\alpha), \varphi(\beta) \rangle = \langle \varphi(s_{\alpha_{i_r}} \dots s_{\alpha_{i_1}}(\alpha)), \alpha'_j \rangle.$$

By Step 1 we get  $\langle \varphi(\alpha), \varphi(\beta) \rangle = \langle s_{\alpha_{i_r}} \dots s_{\alpha_{i_1}}(\alpha), \alpha'_j \rangle$ . By moving the reflections back to the right-hand side we get

$$\langle \varphi(\alpha), \varphi(\beta) \rangle = \langle \alpha, s_{\alpha_{i_1}} \dots s_{\alpha_{i_r}}(\alpha_j) \rangle = \langle \alpha, \beta \rangle$$

as required.

The rest of the proof proceeds as before. The displayed equation on page 123 has already been established in Step 2.

p123, Exercise 11.12 (†) Add the hypotheses that  $n \geq 2$  and that  $U_1, U_2, \dots, U_n$  are distinct.

p126, Before first displayed equation Insert ‘A case by case analysis shows the weight 0 does not arise in this action.’ before ‘Let’ on line 7.

p126, 2nd displayed equation Should read  $\Phi = \{\alpha \in H^* : \alpha \neq 0, L_\alpha \neq 0\}$ .

p126, Equation (★) (†) Replace  $L_0$  with  $H$ .

p126, Lemma 12.2 (†) Replace ‘Suppose that for all  $h \in H \dots$ ’ with ‘so for all  $\alpha \in \Phi$  there is some  $h \in H$  such that  $\alpha(h) \neq 0$ ’. (This follows from (★), since by assumption  $L$  is a classical Lie algebra, and so the off-diagonal part of  $L$  is, by (★), a sum of non-zero weight spaces.)

p127, Prop. 12.3 Replace ‘...where  $\Phi$  is the set of  $\alpha \in H^*$  such that ...’ with ‘...where  $\Phi$  is the set of non-zero  $\alpha \in H^*$  such that ...’.

p131, Step (2c) Change  $h_{ij} = \dots$  to  $k_{ij} := \dots$  (twice).

p132, Step (4) ( $\dagger$ ) Replace  $h_\beta$  with  $h_{\beta_\ell}$ .

p134, After Dynkin diagram ( $\dagger$ ) Replace  $\mathfrak{sl}(3, \mathbf{C})$  with  $\mathfrak{sl}(4, \mathbf{C})$ .

p135, Step (1) The definitions of  $p_{ii}$  and  $q_{ii}$  should be given separately, as  $p_{ii} = e_{i,\ell+i}$ ,  $q_{ii} = e_{\ell+i,i}$ .

p135, Step (2) Replace ‘If  $\alpha = \varepsilon_i + \varepsilon_j \dots$ ’ with ‘If  $\alpha = \varepsilon_i + \varepsilon_j$ , then  $x_\alpha = p_{ij}$  and  $x_{-\alpha} = q_{ji}$  and

$$h = (e_{ii} - e_{\ell+i,\ell+i}) + (e_{jj} - e_{\ell+j,\ell+j})$$

if  $i \neq j$ , and  $h = e_{ii} - e_{\ell+i,\ell+i}$  if  $i = j$ . Hence  $[h, x_\alpha] = 2x_\alpha$  in both cases.’

p136, Step (4) ( $\dagger$ ) In second displayed equation replace  $i = \ell - 1$  with  $j = \ell - 1$ .

p144, proof of Lemma 13.6 ( $\dagger$ ) Replace the final two lines, which do not rule out multiple edges, with ‘Thus the number of edges incident to  $v$  is  $\sum_{i=1}^k 4(v, v_i)^2 < 4$ .’

p145, Lemma 13.9 ( $\dagger$ ) Replace ‘The graph  $\Gamma$  has’ with ‘If the graph  $\Gamma$  is connected then  $\Gamma$  has’.

p148, proof of Proposition 13.2 ( $\dagger$ ) Replace ‘if  $q = 1$ , then there is no restriction on  $r$ ’ with ‘if  $q = 1$  then there is no restriction on  $p$ ’. (We already know that  $r = 1$ .)

p151, Exercise 13.1 ( $\dagger$ ) Replace ‘That is, find a linear map between the vector spaces …’ with ‘That is, find a linear map between the underlying vector spaces of root systems of types  $B_2$  and  $C_2$  …’.

p154, proof of Prop. 14.2 ( $\dagger$ ) In penultimate paragraph, replace ‘we only have to show  $[x, a] \in L \dots$ ’ with ‘we only have to show  $[x, a] \in I \dots$ ’.

p156, penultimate line of proof ( $\dagger$ ) Replace  $k = \langle \gamma, \alpha_i \rangle$  with  $k = -\langle \gamma, \alpha_i \rangle$ .

p160, Exercise 14.3 ( $\dagger$ ) If the field has characteristic two then it is correct to instead assume  $[x, x] = 0$  for all  $x \in L$ . The Lie algebra generated by  $x$  and  $y$  is, by definition, the smallest Lie subalgebra of  $L$  containing  $x$  and  $y$ .

p164, line 7 of §15.1 Replace ‘with respect to  $\Phi$ ’ with ‘with respect to  $\Pi$ ’.

p164, line 2 of §15.1.1 ( $\dagger$ ) Replace ‘on  $L$ ’ with ‘on  $V$ ’.

p166, proof of Lemma 15.3 Here  $\{\alpha_1, \dots, \alpha_\ell\}$  is a base of a root system for the Lie algebra  $L$ . The subspace  $W$  should be defined as the span of elements of the form

$$f_{\alpha_{i_1}} f_{\alpha_{i_2}} \dots f_{\alpha_{i_k}} \cdot v$$

and not as the span of the single element

$$f_{\alpha_1} f_{\alpha_2} \dots f_{\alpha_k} \cdot v.$$

Similar changes must then be made in the following lines. See also the next correction.

p166, proof of Lemma 15.3 In the second displayed equation replace  $[e_\alpha, f_{\alpha_1}]$  with  $[e_\alpha, f_{\alpha_{i_1}}]$ .

p167, line 3 (†) Replace  $\varepsilon_3$  with  $-\varepsilon_3$ .

p168, 2nd displayed equation Change  $v_i \wedge w_j$  to  $v_i \wedge v_j$ .

p170, §15.1.3 Replace ‘we introduce a symbol  $v_i \otimes w_j$ ’ with ‘we introduce a symbol  $v_i \otimes w_j$ ’.

p174, §15.2.2 (†) Line 8 of §15.2.2: replace  $\alpha \in \Phi$  with  $\alpha \in \Phi^+$ .

p176, (2) line 2 (†) Replace  $M(\lambda)$  with  $M(0)$ .

p178, before Exercise 15.4 The definition of the Chevalley group should read ‘ $G_F(L) := \langle \tilde{A}_\alpha(t) : \alpha \in \Phi, t \in F \rangle$ ’.

p178, line 3 from bottom (†) Replace ‘field with 2-elements’ with ‘field with 2 elements’.

p188, Exercise 15.10 (†) Replace  $\nu(x)$  with  $\nu x$ .

p183, penultimate paragraph Replace ‘prime characteristic 0’ with ‘prime characteristic  $p$ ’.

p184, diagram There is an extra ‘3’ in the diagram of the quiver.

p187, Exercise 15.8 Make the following addition: ‘... if we replace  $\iota$  with  $\iota'$  and  $U(L)$  with  $V$  in the commutative diagram above, then  $V$  has the universal property ...’.

p191, Theorem 16.1(a) Replace  $V/\ker \alpha \cong W$  with  $V/\ker \alpha \cong \text{im } \alpha$ .

p196, second displayed equation Replace  $a(X)(x - \lambda_i)^{a_i} v$  with  $a(x)(x - \lambda_i)^{a_i} v$ . Replace  $f(X)$  with  $p(X)$  in last line.

p196, line 7 (†) Insert a comma after  $(X - \lambda_1)^{a_1}$ .

p198, diagram The vectors spanning the axes must be swapped.

- p202, final paragraph The notation could perhaps make it clearer that  $W$  is a subspace of  $V^*$ , not of  $V$ .
- p210, final line Replace  $a_{ti}$  with  $a_{it}$  in the displayed equation.
- p212, lines 2 and 3 (†) Replace  $a_{ji}$  with  $-a_{ij}$ .
- p213, penultimate line (†) The identity map on  $V$  does not have image in  $W$ , so is not an admissible choice. Instead take a subspace  $U$  of  $W$ , so that  $V = W \oplus U$  as vector spaces, and take the map  $g : V \rightarrow W$  that is the identity on  $W$  and has kernel  $U$ . The coset  $g + M_0$  is then a non-zero element of  $M_S/M_0$ .
- p219, (†) Begin the proof by ‘Define  $m = \dim V_0$ .’ Then at the end of the first paragraph, change  $m = 1$  to  $m = 2$ .
- p219, proof of Corollary 18.4 (†) Replace  $P^{-t}SP^{-1}$  with  $P^{-1}SP^{-t}$  in the first and second lines and  $S = P^tSP$  with  $S = PSP^t$  in the third line.
- p221, displayed equations (†) Replace  $\mu_m y_m$  with  $\mu_{m-1} y_{m-1}$  in the first and  $\mu_m(\text{ad } t)y_m$  with  $\mu_{m-1}(\text{ad } t)y_{m-1}$  in the second.
- p227, paragraph 2 (†) In final line replace  $s_{\beta_n} \in N$  with  $s_{\beta_\ell} \in N$ .
- p227, paragraph 4 (†) In penultimate line of the proof replace  $s_{\alpha_{n-1}}N$  with  $s_{\alpha_{\ell-1}}N$ . In final line replace  $\{1 \dots n\}$  with  $\{1, \dots, \ell\}$ .
- p227, final line (†) Delete ‘below’.
- p228, first displayed equation (†) Replace  $s_{\alpha_{n-1}}s_{\beta_n}$  with  $s_{\alpha_{\ell-1}}s_{\beta_\ell}$ .
- p228, Exercise 19.2.\* (†) The ‘only if’ direction is simply false: for example take  $v_1 = v_2 \neq 0$ . It holds if, in addition, we have  $(v_i, v_j) < 0$  for all  $i$  and  $j$ , as when  $v_1, \dots, v_k$  are part of a base of a root system; then it can be proved by the argument in the proof of Theorem 11.9 (not Theorem 11.10).
- p231, Line –3 (†) Replace 1.13 with 1.12.
- p233, 2.11 In first line replace  $PxP^{-1}$  with  $P^{-1}xP$ .
- p234, 3.2 Replace  $\varphi[y_1z_1]$  with  $\varphi[y_1, z_1]$ .
- p236, 6.5 Replace ‘there is a basis of  $\text{ad } L \dots$ ’ with ‘there is a basis of  $L \dots$ ’
- p238, 8.6(i) In second display, the second line should end  $\frac{1}{2}h(eh + 2e) \cdot v$ , not  $\frac{1}{2}h(he + 2e) \cdot v$ .

p240, 9.15 (†) In fourth paragraph of solution, change ‘generated by  $w$ ’, to ‘generated by  $w$ , where  $w$  is any vector such that  $w + M = v$ ’.

p241, 10.6 Displayed equation should end  $\dots = -1 \times -1 = 1$ .

p241, 11.9 (†) This solution should appear before the solution to Exercise 11.12. A simpler solution uses that since  $M$  is invertible, there exists a unique  $v \in \mathbf{R}^n$  such that  $Mv = (1, \dots, 1)^t$ . The claim then follows taking  $z = v_1 b_1 + \dots + v_n b_n$ .

p246, 19.1 (†) In second line  $q_{1i_1}$  should read  $q_{i_1 1}$ .