The Liar Game

Mark Wildon
Guessing Games

Ask a friend to think of a number between 0 and 15. How many NO/YES questions do you need to ask to find out the secret number?
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Q1
Is the number 8, 9, 10, 11, 12, 13, 14 or 15?

YES
Guessing Games

Ask a friend to think of a number between 0 and 15. How many NO/YES questions do you need to ask to find out the secret number?

Q1
Is the number 8, 9, 10, 11, 12, 13, 14 or 15?

Q2
Is the number 12, 13, 14 or 15?
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Q1
Is the number 8, 9, 10, 11, 12, 13, 14 or 15?

Q2
Is the number 12, 13, 14 or 15?

Q3
Is the number 14 or 15?

Q4
Is the number 13?
Mathematicians like to give rigorous arguments to justify the claims they make.

It is believable that no questioning strategy can guarantee to find the secret number using three or fewer questions. But can we prove this beyond any doubt?
Proofs

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▶ At the start of the game, there are 16 possible numbers.
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- Suppose that if the first question is answered ‘YES’ then there are $r$ possible numbers.
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- Suppose that if the first question is answered ‘YES’ then there are $r$ possible numbers.
  - Then there are $16 - r$ possible numbers if the first question is answered ‘NO’.
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- Suppose that if the first question is answered ‘YES’ then there are $r$ possible numbers.
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  - Either $r$ or $16 - r$ is at least 8.
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  - Hence, in the worst case, there are at least 8 possible numbers after Question 1.
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- In the worst case, there are at least 4 possible numbers after Question 2.
Proofs

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It is believable that no questioning strategy can \textbf{guarantee} to find the secret number using three or fewer questions. But can we prove this \textbf{beyond any doubt}? 

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  - Hence, in the worst case, there are at least 8 possible numbers after Question 1.
- In the worst case, there are at least 4 possible numbers after Question 2.
- In the worst case, there are at least 2 possible numbers after Question 3.
Binary and Computers

In a modern computer, everything is stored as a lists of the bits (binary digits) 0 and 1.
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Books, music, videos, computer programs, bitcoins . . . , are all stored as bits.
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01110000 11010111 00000100 00010101 11010100 01000110 00000100 11010111
00110101 00000100 01101111 11010111 00001000 01101000 01110101 11010111
00000100 00101011 11010100 00100111 00000100 01110100 00010110 10010100
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01010111 00100111 00000111 11110001 00101100 11010111 00010110 11110100
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William Shakespeare (approx 1600)
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11010100 01000110
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_To be, or not to be: that is the question:_
_Whether 'tis nobler in the mind to suffer_
_The slings and arrows of outrageous fortune,_
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Anonymous Microsoft Programmer (2010?)

Part of the machine code for Microsoft Word 2011.
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Why Coding Theory?

A bit gives a single piece of information: ‘NO’ or ‘YES’; ‘on’ or ‘off’; 0 or 1.

- A number between 0 and 15: 4 bits
- A small QR-code:
- Text on this slide
- Full text of *Hamlet*
- Pictures of Royal Holloway
- Compact disc of Beethoven 9th
- Bluray disc of 3 hour film
- Large Hadron Collider, per second
Why Coding Theory?

A bit gives a single piece of information: ‘NO’ or ‘YES’; ‘on’ or ‘off’; 0 or 1.

- A number between 0 and 15: 4 bits
- A small QR-code: 441 bits
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- Text on this slide 10000 bits
- Full text of *Hamlet* 1.5 million bits
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- A number between 0 and 15: 4 bits
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- Text on this slide: 10000 bits
- Full text of *Hamlet*: 1.5 million bits
- Pictures of Royal Holloway: 5 million bits each
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- Text on this slide: 10000 bits
- Full text of *Hamlet*: 1.5 million bits
- Pictures of Royal Holloway: 5 million bits each
- Compact disc of Beethoven 9th: 0.7 GB
- Bluray disc of 3 hour film
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Errors in reading and writing are inevitable. The 3G specification for mobile phones expects one bit in a thousand to be received wrongly.
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The Mariner 9 probe, launched in 1971, took the first images of Mars. The images were grey-scale, with 64 possible shades of grey for each pixel.

- The pictures were transmitted back to Earth by sending one pixel at a time. Since $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$, each pixel needs 6 bits to send.
- The probability of each bit being flipped in the channel was about 5%.
- Encoding each pixel using 6 bits, about 26% of every image would be wrong.
Mariner 9

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- The probability of each bit being flipped in the channel was about 5%.
- Encoding each pixel using 6 bits, about 26% of every image would be wrong.
- Instead each pixel was encoded using 32 bits, increasing the length of the transmitted message over five times.
Mariner 9 Image: Improvement Due to Error Correction
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The Mariner 9 Code: 32 of the 64 Mariner 9 codewords

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\end{array}
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The Mariner 9 Code: 32 of the 64 Mariner 9 codewords
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It is not compulsory to lie.
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Any interesting strategies?
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It is not compulsory to lie.

Any interesting strategies? (For example, asking if someone has lied so far?)
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**Claim.** No strategy can guarantee to use fewer than 7 questions.
Ask a friend to think of a number between 0 and 15. How many NO/YES questions do you need to ask, if your friend is permitted to lie at most once?

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Claim. No strategy can guarantee to use fewer than 7 questions.

Coding theory can be used to find a good strategy. Lies correspond to errors in transmission.
Richard Hamming discovered a one-error correcting binary code of length 7 with 16 codewords. He invented it because he was fed up with the paper tape reader on his early computer misreading his programs.

It gives an optimal solution to the Liar Game using 7 questions.

Remarkably, it is possible to specify all the questions in advance.
The Hamming Code

Find the binary codeword corresponding to your secret number.

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<td>15</td>
<td>1111111</td>
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The questions are:

‘Is there a 1 in the first position (far left) of the codeword?’,
‘Is there a 1 in the second position of the codeword?’,
and so on. If there is one lie, then the questioner will write down one wrong bit. But because the Hamming code can correct one error, the questioner can still work out what the number is.
The Square Code

To encode a number between 0 and 15 in the square code

- Write it in binary as $b_1b_2b_3b_4$
- Make a square with these bits.
- Put in four check bits around the edges, computed using modulo 2 arithmetic:

$$0 + 0 = 0, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 1 + 1 = 0.$$
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Imagine you receive this.

\[
\begin{array}{ccc|c}
0 & 0 & 1 \\
1 & 1 & 0 \\
\hline
1 & 0 \\
\end{array}
\]

What number do you think was sent?

(A) 3 = 0011  (B) 5 = 0101  (C) 6 = 0110  (D) 7 = 0111
Square Code
Square Code
Double Square Code (16 bits) Versus No Coding (4 bits)
Double Square Code (16 bits) Versus No Coding (4 bits)
A Hat Game Related to Coding Theory

You and two friends are on your way to a party.

At the party a black or blue hat will be put on each person’s head. You can see your friends’ hats, but not your own.
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When the host shouts ‘go!’ you may either say a colour or remain silent. Everyone who speaks must speak at the same time.

If everyone who speaks gets the colour of his or her hat correct, you all win some cake. If no-one speaks, or someone gets it wrong, there is no cake.
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**Question:** What is a good strategy?
Another Hat Game

You and four friends are lined up. A black or blue hat is put on each person’s head. You can see all the hats in front of you, but not your own, or those behind.

So the person at the back of the line can see four hats, the next person can see three, and so on.
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Starting at the back of the line, each person is asked to guess the colour of his or her hat.
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**Question:** What is a good strategy?
Thank you! Any questions?
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- Why is maths a good subject to study?
- What do maths lecturers do all day?
- How does maths at university differ from A-level maths?
- Are women just as good as men at maths? (Answer: Yes!)