.

PERIODIC COHOMOLOGY AND SUBGROUPS WITH BOUNDED BREDON COHOMOLOGICAL DIMENSION

JANG HYUN JO* AND BRITA E. A. NUCINKIS

ABSTRACT. Mislin and Talelli showed that a torsion-free group in $\mathbf{H}\mathfrak{F}$ with periodic cohomology after some steps has finite cohomological dimension. In this note we look at similar questions for groups with torsion by considering Bredon cohomology. In particular we show that every elementary amenable group acting freely and properly on some $\mathbb{R}^n \times S^m$ admits a finite dimensional model for $\underline{E}G$.

1. INTRODUCTION

Let G be a discrete group. If G acts freely and properly on $\mathbb{R}^n \times S^m$, then it has periodic cohomology after some steps, i.e., there are positive integers q and k such that $H^i(G, -)$ and $H^{i+q}(G, -)$ are naturally equivalent for all $i > k$. Since $\mathbb{R}^n \times S^m$ is a separable metric space, these groups are necessarily countable. It is conjectured that for countable groups these two conditions are equivalent. Talelli recently proved this for countable groups belonging to Kropholler's class $\mathbf{H}\mathfrak{F}$ [19].

A group G of finite cohomological dimension is torsion-free and has trivial periodic cohomology after k steps, where $k = \operatorname{cd} G$. Since there is no known example of a torsion-free group G of infinite cohomological dimension with periodic cohomology, Talelli posed the following conjecture:

Conjecture 1. (Talelli [18]) *A torsion-free group with periodic cohomology after some steps has finite cohomological dimension.*

It was shown by Mislin and Talelli [13, Theorem 4.9 (i)] that this conjecture holds for groups belonging to $\mathbf{H}\mathfrak{F}$.

It is natural to ask how these questions relate to groups with torsion. A G-CW-complex X is called a classifying space for proper actions, or $\underline{E}G$, if X^H is contractible for all finite subgroups $H < G$ and empty otherwise. For a torsion-free group a model for $\underline{E}G$ is the same as an EG, the universal cover of a $K(G, 1)$. The algebraic mirror to this is Bredon cohomology. It was shown by Lück [9] that a group admits a finite dimensional model for $\underline{E}G$ if and only if it has finite Bredon cohomological dimension, $\underline{\operatorname{cd}}\, G$. There

Date: 6 June 2006.

2000 *Mathematics Subject Classification.* 20J05, 18G20.

Key words and phrases. Bredon cohomology, Elementary amenable group, Hirsch length.

$*$ The author was supported by Korea Research Foundation Grant (KRF-2004-075-C00003)(2004).

is an example which supports our curiosity: every $\mathbf{H}\mathfrak{F}$-group of type FP_∞ admits a finite dimensional model for $\underline{\mathrm{E}}\mathrm{G}$ [8]. Thus we may naturally ask the following question:

Question 1. *Does every group acting freely and properly on $\mathbb{R}^n \times S^m$ admit a finite dimensional model for $\underline{\mathrm{E}}\mathrm{G}$?*

We give a positive answer to this question for elementary amenable groups in section 4.
If a torsion-free group G admits a finite dimensional model for $\mathrm{E}G$, then there is obviously a free G-CW-complex homotopy equivalent to a sphere. If, in addition, G is countable, [13, Lemma 5.4] implies that G acts freely and properly on some $\mathbb{R}^n \times S^m$. Hence the following question arises.

Question 2. *Let G be a group admitting a finite dimensional model for $\underline{\mathrm{E}}\mathrm{G}$ such that all finite subgroups have periodic cohomology. Does G act freely on a G-CW-complex homotopy equivalent to a sphere?*

The answer is yes, if we also assume that G has a bound on the orders of the finite subgroups, which follows from [13, Theorem B]. On the other hand, the infinite locally cyclic p-group C_p^∞, given by

$$C_p^\infty = \langle c_1, c_2, ..., c_i, ... \,|\, c_2^p = c_1, ..., c_{i+1}^p = c_i, ... \rangle$$

admits a 1-dimensional model for $\underline{\mathrm{E}}\mathrm{G}$ and has periodic cohomology of period 2 after one step [17, Theorem 3.5]. A recent result of O. Talelli [19, Corollary 3.5] implies that G acts freely on a G-CW-complex homotopy equivalent to a sphere. Since it is countable, it acts freely and properly on some $\mathbb{R}^n \times S^m$. We do not know of a general answer to Question 2.

If G has periodic cohomology after k steps, then so does every subgroup $H < G$ of finite cohomological dimension and hence $\operatorname{cd} H \leq k$. This naturally leads to the following question on cohomological dimensions alone.

Question 3. (Talelli [1]) *Is there a torsion-free group G of infinite cohomological dimension with the following property: there exists N such that for all proper subgroups H with finite cohomological dimension $\operatorname{cd} H \leq N$?*

Groups satifying the condition that there exists an N with the property that if H is a proper subgroup of G with finite cohomological dimension $\operatorname{cd} H$, then $\operatorname{cd} H \leq N$, are said to have *jump cohomology.* Petrosyan [15] showed that torsion free $\mathbf{H}\mathfrak{F}$-groups with jump cohomology have finite cohomolgical dimension.

In light of Question 1, it is natural to translate the above questions for torsion-free groups to Bredon cohomology.

Question 4. *Does the condition that G has periodic Bredon cohomology after some steps imply that $\underline{\operatorname{cd}}\, G < \infty$?*

The adjoint isomorphism for Bredon-cohomology [9] and the fact that any model for $\underline{\mathrm{E}}\mathrm{G}$ is also a model for any subgroup of G, gives us the Bredon-version of the Eckmann-Shapiro Lemma and hence, periodic Bredon cohomology after some steps is a subgroup-closed property. Analogous to the ordinary case it thus makes sense to ask the following question.

Question 5. *Suppose that every proper subgroup H of G with finite Bredon cohomological dimension satisfies $\underline{\mathrm{cd}}\,H \leq N$ for some N. Is $\underline{\mathrm{cd}}\,G < \infty$?*

Tying in with the above is recent work of Talelli [20] on groups of type Φ. A group is of type Φ if for every $\mathbb{Z}G$-module M, $proj.dim_{\mathbb{Z}G}M < \infty$ if and only if $proj.dim_{\mathbb{Z}H}M < \infty$ for all finite subgroups $H < G$. This property is closely related to the invariants $\mathrm{silp}\,G$, the supremum of the injective dimensions of the projectives, $\mathrm{spli}\,G$, the supremum of the projective dimensions of the injectives and $\mathrm{fin.dim}\,G = sup\{proj.dim_{\mathbb{Z}G}M \,|\, proj.dim_{\mathbb{Z}G}M < \infty\}$. Talelli proposed the following conjecture:

Conjecture 2. (Talelli [20]) *The following statements are equivalent for a group G.*

1. *G admits a finite dimensional model for* $\underline{\mathrm{E}}\mathrm{G}$
2. *G is of type Φ*
3. $\mathrm{spli}\,G < \infty$
4. $\mathrm{silp}\,G < \infty$
5. $\mathrm{fin.dim}\,G < \infty$,

and showed the implications (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) and that the implication (5) $\Rightarrow$ (1) holds for torsion-free locally soluble groups [20, Corollary 3.6]. It follows from [20, Lemma 3.5] that any group of finite finitistic dimension has jump cohomology.

The purpose of this paper is to study the above conjectures and questions for elementary amenable groups. We shall show that every countable elementary amenable group with jump cohomology admits a finite dimensional model for $\underline{\mathrm{E}}\mathrm{G}$. We shall also consider the behaviour of groups with periodic Bredon cohomology under extensions. We shall use a purely algebraic approach considering the behaviour of the Bredon homological dimension, $\underline{\mathrm{hd}}\,G$, of our groups. As in ordinary cohomology, $\underline{\mathrm{cd}}\,G$ and $\underline{\mathrm{hd}}\,G$ are closely related. It was shown [14] that $\underline{\mathrm{hd}}\,G \leq \underline{\mathrm{cd}}\,G$, and that for countable groups $\underline{\mathrm{cd}}\,G \leq \underline{\mathrm{hd}}\,G + 1$. Flores and the second author [3] also showed that for elementary amenable groups the Hirsch length $\mathrm{h}G$ is equal to $\underline{\mathrm{hd}}\,G$. It therefore makes sense to address Question 5 in terms of the Hirsch length.

The paper is organized as follows. In section 2, we briefly recall the definitions and some basic facts about Bredon cohomology, elementary amenable groups and Hirsch length. In Sections 3 and 4 we shall address the above questions and conjectures for elementary amenable groups and deal with related questions on group extensions.

While working on this paper the authors met at the ETH, Zürich. The first author gratefully acknowledges the support and hospitality of the ETH.

2. Preliminaries

Let $\mathfrak{F}$ be a family of subgroups of G. The orbit category $\mathcal{O}_{\mathcal{F}}(G)$ is the category whose objects are the G-sets G/H for any finite subgroup H of G and morphisms are the G-maps $G/H_1 \to G/H_2$. We write $\mathrm{Mod}_{\mathcal{F}}$-$G$ for the category of contravariant functors from $\mathcal{O}_{\mathcal{F}}(G)$ to $\mathfrak{Ab}$, the category of abelian groups. This is an abelian category with enough projectives. Define $\mathrm{cd}_{\mathfrak{F}}G$ as the projective dimension of $\underline{\mathbb{Z}}$ in $\mathrm{Mod}_{\mathcal{F}}$-$G$, the constant functor, and $\mathrm{hd}_{\mathfrak{F}}G$ as the flat dimension of $\underline{\mathbb{Z}}$ in $\mathrm{Mod}_{\mathcal{F}}$-$G$. If $\mathfrak{F}$ is the family of finite subgroups of G, then we denote $\underline{\mathrm{cd}}\,G = \mathrm{cd}_{\mathfrak{F}}G$ and $\underline{\mathrm{hd}}\,G = \mathrm{hd}_{\mathfrak{F}}G$. As mentioned before, we have the following properties:

Proposition 2.1. [14] *Let G be a group.*

(1) $\underline{\mathrm{hd}}\,G \leq \underline{\mathrm{cd}}\,G$.
(2) *If G is countable, then* $\underline{\mathrm{cd}}\,G \leq \underline{\mathrm{hd}}\,G + 1$.

For classes of groups $\mathfrak{A}$ and $\mathfrak{B}$, let $\mathfrak{A}\mathfrak{B}$ denote the class of $\mathfrak{A}$-by-$\mathfrak{B}$ groups, that means the class of groups G, for which there is an exact sequence of groups $1 \to N \to G \to Q \to 1$ with $N \in \mathfrak{A}$ and $Q \in \mathfrak{B}$. Similarly, let $L\mathfrak{A}$ denote the class of locally $\mathfrak{A}$-groups. Denote by $\mathfrak{X}$ the class of elementary amenable groups. This is the smallest class of groups, which contains all abelian and all finite groups and is closed under extensions and directed unions. In [6], the following alternative description of the class of elementary amenable groups was given. Let $\mathfrak{Y}$ denote the class of all finitely generated abelian-by-finite groups. For each ordinal α, we define the class $\mathfrak{X}_\alpha$ inductively as follows:

$$\mathfrak{X}_0 = \{1\},$$
$$\mathfrak{X}_\alpha = (L\mathfrak{X}_{\alpha-1})\mathfrak{Y} \text{ if } \alpha \text{ is a successor ordinal,}$$
$$\mathfrak{X}_\alpha = \cup_{\beta<\alpha}\mathfrak{X}_\beta \text{ if } \alpha \text{ is a limit ordinal.}$$

The class of all elementary amenable groups is defined by:

$$\mathfrak{X} = \bigcup_{\alpha \geq 0} \mathfrak{X}_\alpha.$$

For a soluble group G the Hirsch length $\mathrm{h}G$ is the sum of the torsion-free ranks of the abelian factors in the derived series of G. Hillman [4] extended the notion of Hirsch length of a soluble group to the class of elementary amenable groups. Notice that a group is elementary amenable of Hirsch length 0 if and only if it is locally finite and Hirsch length is additive for an extension of groups. It is a well known result by Stammbach [16, 2] that for a torsion free soluble group the Hirsch length is equal to the homological dimension over $\mathbb{Q}$. This was also shown to be true for elementary amenable groups [4] and was further extended by Flores and the second author as follows:

Proposition 2.2. [3] *Let G be an elementary amenable group. Then* $\mathrm{h}G = \underline{\mathrm{hd}}\,G$.

The class $\mathbf{H}\mathfrak{F}$ of hierarchically decomposable groups was introduced by Kropholler [7] as follows. Let $\mathbf{H}_0\mathfrak{F}$ be the class of finite groups. Now define $\mathbf{H}_\alpha\mathfrak{F}$ for each ordinal α inductively: if α is a successor ordinal, then $\mathbf{H}_\alpha\mathfrak{F}$ is the class of groups G which admit a finite dimensional contractible G-CW-complex with isotropy groups in $\mathbf{H}_{\alpha-1}\mathfrak{F}$, and if α is a limit ordinal, then $\mathbf{H}_\alpha\mathfrak{F} = \cup_{\beta<\alpha}\mathbf{H}_\beta\mathfrak{F}$. A group belongs to $\mathbf{H}\mathfrak{F}$ if it belongs to $\mathbf{H}_\alpha\mathfrak{F}$ for some α. The class $\mathbf{H}\mathfrak{F}$ contains all elementary amenable groups, all linear groups, and all groups of finite virtual cohomological dimension. It is subgroup closed, extension closed, and closed under arbitrary directed unions.

3. Subgroups with bounded Bredon cohomological dimension

Proposition 3.1. *Let G be an elementary amenable group for which there exists a positive integer N, such that for each proper subgroup $H < G$ of finite Bredon-cohomological dimension, $\underline{\mathrm{cd}}\, H \leq N$. Then $\underline{\mathrm{hd}}\, G < \infty$. If, in addition, G is countable, then also $\underline{\mathrm{cd}}\, G < \infty$.*

Proof. Let $\mathcal{C}$ denote the class of groups for which there exists a positive integer N, such that for each subgroup $H < G$ of finite Bredon-cohomological dimension, $\underline{\mathrm{cd}}\, H \leq N$. Clearly $\mathcal{C}$ is subgroup-closed. Since for elementary amenable groups $\underline{\mathrm{hd}}\, G = \mathrm{h}G$ [3], it suffices to show the result for finitely generated groups G. Let G be a finitely generated elementary amenable group belonging to $\mathcal{C}$. Hence it suffices to show that $\mathrm{h}G < \infty$. We proceed by induction on the height α of G. For $\alpha = 0$, there is nothing to prove. Let $\alpha > 0$ and assume that the claim is true for all $H \in \mathfrak{X}_\beta$ with $\beta < \alpha$. We might also assume that α is a successor ordinal. Consider an extension

$$H \hookrightarrow G \twoheadrightarrow Q,$$

where H lies in $L\mathfrak{X}_{\alpha-1}$ and Q is finitely generated abelian-by-finite. It is obvious that Q has finite Hirsch length hQ and H is the direct limit of groups H_i lying in $\mathfrak{X}_{\alpha-1}$. Notice that each H_i lies in $\mathcal{C}$ and hence by induction has finite Hirsch length. Since each H_i is also a subgroup of G with finite Hirsch length, the fact that G belongs to $\mathcal{C}$ implies that $\mathrm{h}H_i \leq N$ for all i. Thus

$$\mathrm{h}H = \sup\{\mathrm{h}H_i\} \leq N.$$

Hence we have that

$$\mathrm{h}G = \mathrm{h}H + hQ \leq N + hQ < \infty.$$

The result for $\underline{\mathrm{cd}}\, G$ for countable G follows from Proposition 2.1. □

It actually turns out that it suffices to just consider the torsion free subgroups of finite cohomological dimension. In the case of torsion-free $\mathbf{H}\mathfrak{F}$-groups this obviously follows from [15].

Theorem 3.2. *Let G be an elementary amenable group such that there exists a number N such that $\mathrm{cd}\, H \leq N$ for any proper subgroup $H < G$ of finite cohomological dimension, then $\underline{\mathrm{hd}}\, G < \infty$. If, in addition, G is countable, then also $\underline{\mathrm{cd}}\, G < \infty$.*

Proof. Let H be any subgroup of G with finite Bredon cohomological dimension and hence of finite Hirsch-length. By the proof of Proposition 3.1 it suffices to show that $\mathrm{h}H \leq N$. Denote by $\tau(H)$ the unique maximal normal torsion-subgroup of H. There is an extension

$$\tau(H) \rightarrowtail H \twoheadrightarrow Q,$$

where Q is virtually torsion-free soluble [22]. Each torsion-free subgroup of H embeds into Q. Since $\mathrm{h}H = \mathrm{h}Q$ we may assume that H is virtually torsion-free soluble and thereby we have the extension

$$K \rightarrowtail H \twoheadrightarrow F,$$

where K is torsion-free soluble and F is finite. Furthermore, K is torsion free of finite Hirsch length and hence countable [2]. Thus

$$\operatorname{cd} K \leq \mathrm{h}K + 1 = \mathrm{h}H + 1 < \infty.$$

By the assumption, $\mathrm{h}H = \mathrm{h}K = \operatorname{hd} K \leq \operatorname{cd} K \leq N$ as required. □

The following corollary shows that Conjecture 2 holds for countable elementary amenable groups.

Corollary 3.3. *Let G be a countable elementary amenable group. If* $\operatorname{fin.dim} G < \infty$, *then* $\underline{\operatorname{cd}}\, G < \infty$.

Proof. Put $N = \operatorname{fin.dim} G$. From [20, Lemma 3.5], it follows that for any torsion-free subgroup H of finite cohomological dimension, $\operatorname{cd} H \leq N$. Hence $\underline{\operatorname{cd}}\, G < \infty$ by Theorem 3.2. □

4. Periodic Bredon cohomology

We begin by relating periodic cohomology after some steps to jump cohomology in Bredon cohomology.

Lemma 4.1. *Let G have periodic cohomology of period q after k steps. Then for any subgroup $H < G$ of finite Bredon cohomological dimension, $\underline{\operatorname{cd}}\, H \leq k$.*

Proof. Recall that $\underline{\operatorname{cd}}\, H = \inf\{\, n \mid H^i_{\mathcal{F}}(H, -) = 0,\, i > n \,\}$. Suppose that $\underline{\operatorname{cd}}\, H = n > k$. Since $H^{n+q}_{\mathcal{F}}(H,-) = 0$, we deduce that $H^n_{\mathcal{F}}(H,-) = 0$ by periodicity. But this contradicts the minimality of n. □

Theorem 4.2. *Let G be an elementary amenable group with periodic Bredon cohomology after some steps, then $\underline{\operatorname{hd}}\, G < \infty$. If, in addition, G is countable, then also $\underline{\operatorname{cd}}\, G < \infty$.*

Proof. This follows directly from Lemma 4.1 and Proposition 3.1. □

We are now able to give an answer to Question 1 for elementary amenable groups.

Corollary 4.3. *Let G be an elementary amenable group acting freely and properly on $\mathbb{R}^n \times S^m$. Then G has a finite dimensional model for $\underline{\mathrm{E}}G$.*

We will conclude this note with a few remarks on the behaviour of periodic Bredon cohomology under group extensions. We will make use of the spectral sequences for Bredon (co)homology and for classifying spaces for proper actions, see [10, 11]. Let us denote by $\mathfrak{F}(G), \mathfrak{F}(N)$ and $\mathfrak{F}(Q)$ the families of finite subgroup of G, N and Q respectively. Let $\mathfrak{H} = \{S \leq G : N \leq S, S/N \in \mathfrak{F}(Q)\}$. It was shown [11] that there is a natural equivalence of categories

$$\begin{aligned} \mathrm{Mod}_{\mathfrak{H}}\text{-}G &\to \mathrm{Mod}_{\mathcal{F}(Q)}\text{-}Q \\ M &\to \overline{M}, \end{aligned}$$

where $\overline{M}(Q/(S/N)) = M(G/S)$.
Notice also that for every $\mathcal{O}_{\mathfrak{F}}G$-module M, we have a injective module $I(M)$ of M for which $M \rightarrowtail I(M)$ and $H^n_{\mathcal{F}}(G, I) = 0$ for any injective module I and $n \geq 1$ ([9, 12]).

The following theorem can be proved by a similar method to that of Venkov [21, Theorem 3] for ordinary periodic cohomology after some steps (see also [5]).

Theorem 4.4. *Let $1 \to N \to G \to Q \to 1$ be an exact sequence of groups. Suppose that*

(1) $\mathrm{cd}_{\mathfrak{F}(G)\cap S} S \leq n$ *for every* $S \in \mathfrak{H}$
(2) Q *has periodic Bredon cohomology of period* q *after* k *steps.*

Then G has periodic Bredon cohomology of period q after $(n+k)$ steps.

Proof. Consider a short exact sequence

$$0 \to M \to I(M) \to C(M) \to 0,$$

where $C(M)$ is the cokernel. By dimension shifting, we have

$$H^r_{\mathfrak{F}(G)}(G, C^n(M)) \cong H^{r+n}_{\mathfrak{F}(G)}(G, M), r \geq 1,$$

where $C^i(M) = C(C^{i-1}(M))$. Similarly, we have

$$H^r_{\mathcal{F}(S)}(S, C^n(M)) \cong H^{r+n}_{\mathcal{F}(S)}(S, M) = 0, r \geq 1,$$

since $\mathrm{cd}_{\mathfrak{F}(G)\cap S} S \leq n$ for all $S \in \mathfrak{H}$. By [11, Theorem 5.1], we have the following spectral sequence

$$E_2^{p,q} = H^p_{\mathfrak{F}(Q)}(Q, \overline{H^q_{\mathfrak{F}(G)\cap -}(-, C^n(M))}) \Rightarrow H^{p+q}_{\mathfrak{F}(G)}(G, C^n(M)).$$

Notice that this spectral sequence is concentrated on the line $q = 0$. Thus we have the isomorphism

$$E_2^{p,0} = H^p_{\mathfrak{F}(Q)}(Q, \overline{H^0_{\mathfrak{F}(G)\cap -}(-, C^n(M))}) \cong H^p_{\mathfrak{F}(G)}(G, C^n(M)), \ \ p \geq 0.$$

Notice that for $r \geq 1$, we have

$$H^{r+n}_{\mathfrak{F}(G)}(G, M) \cong H^r_{\mathfrak{F}(G)}(G, C^n(M)) \cong H^r_{\mathfrak{F}(Q)}(Q, \overline{H^0_{\mathfrak{F}(G)\cap -}(-, C^n(M))}).$$

Since Q has periodic Bredon cohomology of period q after k steps, we have the desired result. $\square$

Corollary 4.5. *Let $1 \to N \to G \to Q \to 1$ be an exact sequence of groups. Suppose that*

(1) $\underline{\mathrm{cd}}\, N = n < \infty$
(2) *Q has periodic Bredon cohomology of period q after k steps and a bound d on the orders of finite subgroups.*

Then G has periodic Bredon cohomology of period q after $dn + k$ steps.

Proof. For any $S \in \mathfrak{H}$, there is a short exact sequence

$$0 \to N \to S \to S/N \to 0.$$

It follows from [10, Theorem 2.4] that $\underline{\mathrm{cd}}\, S \leq |S/N|\, n$. Thus $\underline{\mathrm{cd}}\, S \leq dn$ for every $S \in \mathfrak{H}$. The result now follows from Theorem 4.4. □

References

[1] M. Bestvina, *Questions in geometric group theory*, http://www.math.utah.edu/~bestvina/eprints/questions-updated.pdf.
[2] R. Bieri, *Homological dimension of discrete groups*, Queen Mary College Mathematics Notes, Second edition, 1982.
[3] R.J. Flores and B.E.A. Nucinkis, *On Bredon homology of elementary amenable groups*, to appear in *Proc. Amer. Math. Soc.*
[4] J.A. Hillman, *Elementary amenable groups and 4-manifolds with Euler characteristic 0*, *J. Austral. Math. Soc. Ser. A* **50** (1991), 160–170.
[5] J.H. Jo, *Multiple complexes and gaps in Farrell cohomology*, *J. Pure Appl. Algebra*, **194** (2004), 147–158.
[6] P.H. Kropholler, P.A. Linnell, and J.A. Moody, *Applications of a new K-theoritic theorem to soluble group rings*, *Propc. Amer. Math. Soc.* **104** (3), (1988), 675–684.
[7] P.H. Kropholler, *On groups of type* $(\mathrm{FP})_\infty$, *J. Pure Appl. Algebra*, **90** (1993), 55–67.
[8] P.H. Kropholler and G. Mislin, *Groups acting on finite dimensional spaces with finite stablizers*, *Comment. Math. Helv.* **73** (1998), 122–136.
[9] W. Lück, *Transformation groups and algebraic K-theory*, Lecture Notes in Mathematics **149**, (Springer, Berlin 1989).
[10] W. Lück, *The type of the classifying space for a family of subgroups*, *J. Pure Appl. Algebra* **149** (2000), 177–203.
[11] C. Martinez-Pérez, *A spectral sequence in Bredon (co)homology*, *J. Pure Appl. Algebra* **176** (2002), 161–173.
[12] G. Mislin, *Equivariant K-homology of classifying spaces for proper actions*, Proper group actions and the Baum-Connes conjecture, 1–78, Adv. Courses Math. CRM Barcelona, Birkhauser, Basel, 2003
[13] G. Mislin and O. Talelli, *On groups which act freely and properly on finite dimensional homotopy spheres*, Computational and geometric aspects of modern algebra (Edinburgh, 1998), London Math. Soc. Lecture Note Ser. **275**, Cambridge Univ. Press, Cambridge, (2000), 208–228.
[14] B.E.A. Nucinkis, *On dimensions in Bredon homology*, *Homology, Homotopy Appl.*, **6**, (2004), no. 1, 33–47.
[15] N. Petrosyan, *Periodicity and jumps in cohomology of R-torsion-free groups*, preprint.
[16] U. Stammbach, *On the weak homological dimension of the group algebra of soluble groups*, *J. London Math. Soc. (2)*, **2** (1970), 567–570.
[17] O. Talelli, *On cohomological periodicity for infinite groups*, *Comment. Math. Helv.* **55** (1980), 85–93.

[18] O. Talelli, *Periodic cohomology and free and proper actions on* $\mathbb{R}^n \times S^m$, *London Math. Soc. Lecture Note Ser., 261, Groups St. Andrews 1997 in Bath, II.*, **261** (1997), 701–717.

[19] O. Talelli, *Periodicity in group cohomology and complete resolutions*, *Bull. London Math. Soc.*, **37** (2005), 547–554.

[20] O. Talelli, *On groups of type* Φ, preprint.

[21] B.B. Venkov, *On homologies of groups of units in division algebras*, *Trudy Mat. Inst. Steklov*, **80** (1965), 66–89. [English transtlation : *Proc. Steklov Inst. Math.*, **80** (1965), 73–100.]

[22] B.A.F. Wehrfritz, *On elementary amenable groups of finite Hirsch number*, *J. Austral. Math. soc. (Series A)*, **58** (1995), 219–221.

Research Institute for Basic Science, Korea University, Seoul 136-701, KOREA
E-mail address: jhjo@sogang.ac.kr

School of Mathematics, University of Southampton, SO17 1BJ, U.K.
E-mail address: B.E.A.Nucinkis@soton.ac.uk